

3.1 - Investigating Angle Relationships

October 11, 2019 1:42 PM

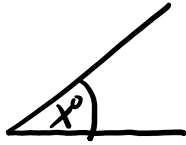
Unit 2: Angles and Triangles

Topic	Assignment
3.1: Investigating Angle Relationships	Pg. 86 # 1 – 9
3.2: Angles Associated with Parallel Lines	Pg. 92 # 1 – 9
3.3: Proving Angle Relationships	Pg. 97 # 3 – 9
3.4: Angle Relationships in Polygons	Pg. 107 # 1 – 10

Review of Angles and Triangles

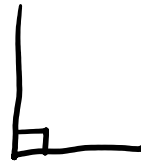
Acute Angle:

Angle "x" measures: $0^\circ \leq x < 90^\circ$



Right Angle:

Angle "x" measures: $x = 90^\circ$



Obtuse Angle:

Angle "x" measures: $90^\circ < x < 180^\circ$



Straight Angle:

Angle "x" measures: $x = 180^\circ$



Reflex Angle:

Angle "x" measures: $180^\circ < x < 360^\circ$



Complete Rotation:

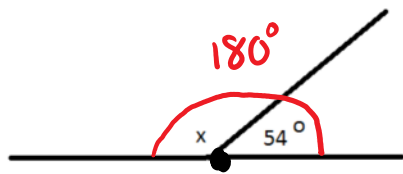
Angle "x" measures: $x = 360^\circ$



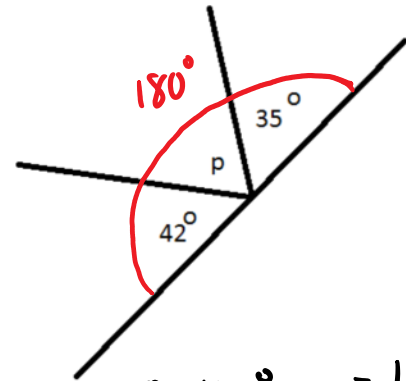
Angles on One Side of a Straight Line

Because we know that a straight line measures 180°, we know all angles on a straight line will add to 180°.

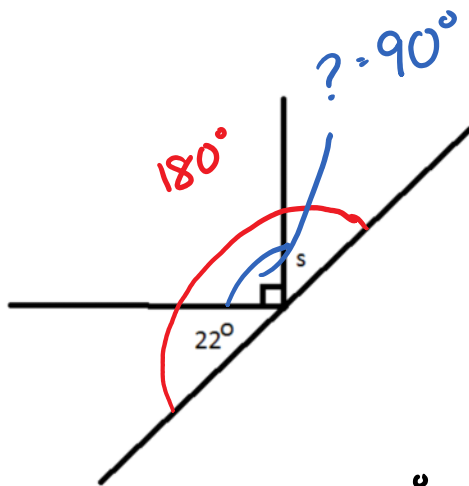
Example. Calculate the following indicated angles:



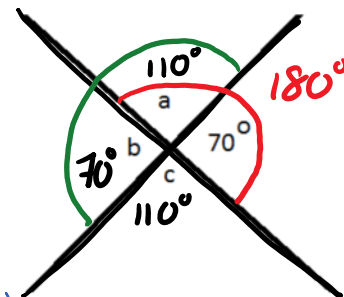
$$\begin{aligned} 54^\circ + x &= 180^\circ \\ -54^\circ & \quad -54^\circ \\ x &= 180^\circ - 54^\circ \\ x &= 126^\circ \end{aligned}$$



$$\begin{aligned} 35^\circ + 42^\circ + p &= 180^\circ \\ 77^\circ + p &= 180^\circ \\ -77^\circ & \quad -77^\circ \\ p &= 180^\circ - 77^\circ = 103^\circ \end{aligned}$$



$$\begin{aligned} 22^\circ + 90^\circ + s &= 180^\circ \\ 112^\circ + s &= 180^\circ \\ -112^\circ & \quad -112^\circ \\ s &= 180^\circ - 112^\circ \\ s &= 68^\circ \end{aligned}$$



$$\begin{aligned} 70^\circ + a &= 180^\circ \\ -70^\circ & \quad -70^\circ \\ a &= 110^\circ \end{aligned}$$

$$\begin{aligned} a + b &= 180^\circ \\ (110^\circ) + b &= 180^\circ \\ -110^\circ & \quad -110^\circ \\ b &= 70^\circ \end{aligned} \quad \left\{ \begin{aligned} b + c &= 180^\circ \\ (70^\circ) + c &= 180^\circ \\ -70^\circ & \quad -70^\circ \\ c &= 110^\circ \end{aligned} \right.$$

Angles which form a straight line are called **supplementary angles**.

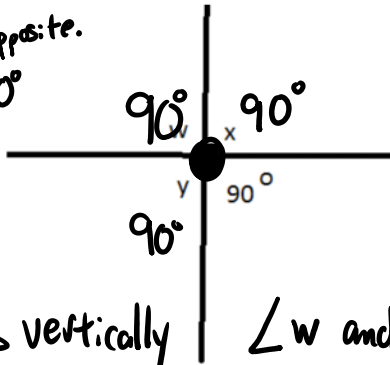
Notice from the last example that the opposite angles are equal.



When straight lines cross and form an "X" shape, the point where they cross is called the "**vertex**", and the angles that are across from each other are called "**vertically opposite angles**", and are equal in measure.

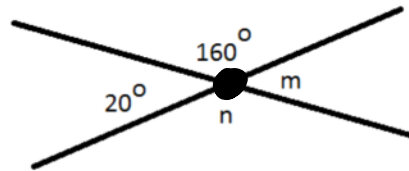
Example: Calculate the indicated angles:

x & y are vertically opposite.
 $\therefore \angle y = 90^\circ$



90° is vertically opposite from angle w ,
 $\therefore w = 90^\circ$

$\angle w$ and $\angle x$ are supplementary,
 so $w + x = 180^\circ$
 $(90^\circ) + x = 180^\circ$
 $-90^\circ \quad -90^\circ$
 $x = 90^\circ$



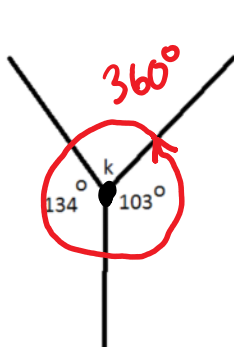
20° is vertically opposite to m , $\therefore m = 20^\circ$,
 160° is vertically opposite to $\angle n$, $\therefore \angle n = 160^\circ$

What do you notice about the sum of all angles in an "X" shape?

They all add to 360° .

Around any vertex or point, all angles will add to 360° , a complete rotation (a complete circle).

Example: Determine the indicated angle:

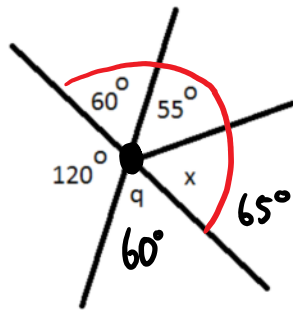


$$k + 134 + 103 = 360^\circ$$

$$k + 237 = 360^\circ$$

$$-237 \quad -237$$

$$k = 360 - 237 = 123^\circ$$



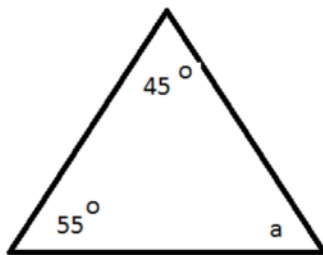
$\angle q$ is vertically opposite 60° ,
 $\therefore \angle q = 60^\circ$

60° and 55° and $\angle x$ are all supplementary,
 $\therefore 60^\circ + 55^\circ + x = 180^\circ$
 $115^\circ + x = 180^\circ$
 -115°
 $x = 65^\circ$

Angles Involving Triangles

The interior angles (inside angles) of a triangle add to 180° :

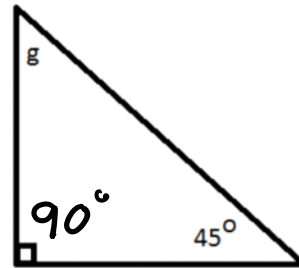
Example: Determine the indicated angles:



$$45^\circ + 55^\circ + a = 180^\circ$$

$$100^\circ + a = 180^\circ$$

$$a = 80^\circ$$

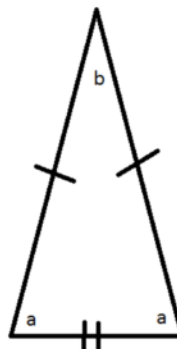


$$90^\circ + 45^\circ + g = 180^\circ$$

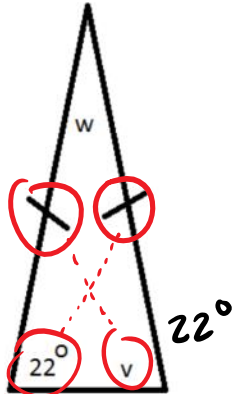
$$135^\circ + g = 180^\circ$$

$$g = 45^\circ$$

An isosceles triangle has 2 equal sides and 2 equal angles:



Example: Determine the indicated angle:

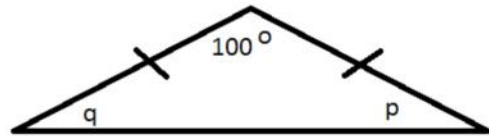


$\angle v = 22^\circ$ \because the measure of angle opposite an equal side length is 22° .

$$22^\circ + 22^\circ + w = 180^\circ$$

$$44^\circ + w = 180^\circ$$

$$w = 136^\circ$$



$$\angle q = \angle p$$

$$q + p + 100^\circ = 180^\circ$$

$$q + q + 100^\circ = 180^\circ$$

$$2q + 100^\circ = 180^\circ$$

$$\begin{array}{r} -100^\circ \\ \hline 2q = 80^\circ \end{array}$$

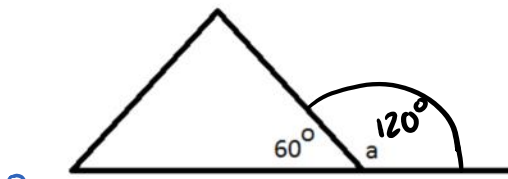
$$\frac{2q}{2} = \frac{80^\circ}{2}$$

$$\because q = p,$$

$$q = 40^\circ \quad p = 40^\circ$$

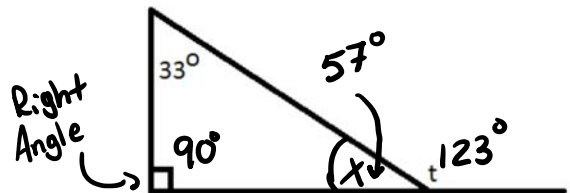
We can use properties of straight lines to find exterior angles of triangles:

Example: Determine the indicated angle:



$$60^\circ + a = 180^\circ$$

$$\begin{array}{r} -60^\circ \\ \hline a = 120^\circ \end{array}$$



$$33^\circ + 90^\circ + x = 180^\circ$$

$$123^\circ + x = 180^\circ$$

$$\begin{array}{r} -123^\circ \\ \hline x = 57^\circ \end{array}$$

$$x = 57^\circ$$

$$57^\circ + t = 180^\circ$$

$$\begin{array}{r} -57^\circ \\ \hline t = 123^\circ \end{array}$$

$$t = 123^\circ$$