3.4 - Reasoning with Polygons

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POLYGONS

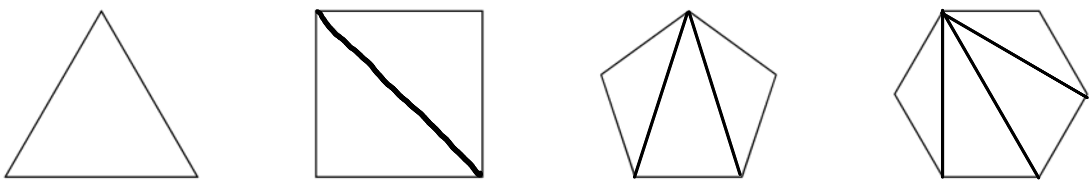
A ***polygon*** is made from line segments joined at their endpoints to make a closed shape; each line segment is a side and the point where the two line segments meet is a ***vertex***. The number of vertices equals the number of sides.

A ***convex polygon*** has interior angles that are all smaller than 180°. A concave polygon will have at least one interior angle larger than 180°.

A ***regular polygon*** sides that are all the same length and interior angles that all have the same measure.

INTERIOR ANGLES OF A POLYGON

Draw diagonals in each polygon, but make the fewest number of triangles.



Complete the table and make a conjecture about the sum of the interior angles.

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	180°
Quadrilateral	4	2	$2(180°) = 360°$
Pentagon	5	3	$3(180°) = 540°$
Hexagon	6	4	$4(180°) = 720°$
Heptagon	7	5	$5(180°) = 900°$
Octagon	8	6	$6(180°) = 1080°$
n-sided polygon	*n*	$n-2$	$(n-2)180$

Conjecture: The interior angles of an *n*-sided polygon add up to $180(n-2)$

where "n" is the number of sides.

exercise: What do the measures of the interior angles in a regular 20-sided polygon add up to? Find the measure of each interior angle.

Sum of internal angles $= 180(n-2)$

$180(20-2) \rightarrow 180(18) = 3240°$

$3240 \div 20$

$= 162°$

Answer: 162°

exercise: The sum of the interior angles of a polygon is 2340°; how many sides does the polygon have?

interior sum $= 180(n-2)$

$\dfrac{(2340°)}{180} = \dfrac{180(n-2)}{180}$

$13 = n-2$

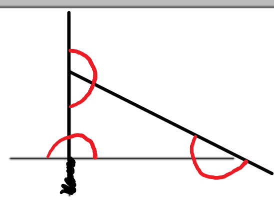
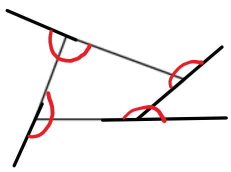
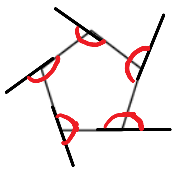
$+2 \quad +2$

$n = 15$

Answer: 15 sided polygon.

EXTERIOR ANGLES OF A POLYGON

Draw external angles for each polygon, complete the table, and make a conjecture about the sum of the exterior angles.

Polygon	Diagram	Sum of Interior & Exterior Angles	Sum of Interior Angles	Sum of Exterior Angles
Triangle		$180° \times 3 = 540°$	180°	$I + E = T$ $180° + E = 540°$ $E = 540° - 180°$ $E = 360°$
Quadrilateral		$180° \times 4 = 720°$	360°	$720° - 360° = 360°$
Pentagon		$180° \times 5 = 900°$	540°	$900° - 540° = 360°$
n-sided polygon		$180° \times n$	$(n-2)180°$	360°

Conjecture: The exterior angles of a convex n-sided polygon add up to $360°$

exercise: For a regular nonagon (9-sided polygon), find:

a) the measure of each exterior angle,

All exterior angles $= 360°$

$360° \div 9$

Answer: $= 40°$

b) the measure of each interior angle.

Sum of interior $= 180(n-2)$

$= 180(9-2)$

$= 180 \times 7$

$= 1260°$

$1260° \div 9$

Answer: $= 140°$

exercise: Each interior angle of a regular polygon is 144°.

a) Find the measure of each exterior angle.

Interior + Exterior $= 180°$

$(144°)$ + Exterior $= 180°$

Exterior $= 36°$

Answer: $36°$

b) How many sides does this polygon have?

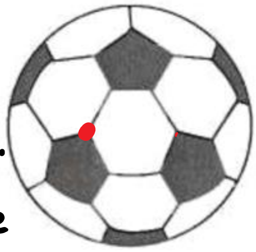
All exterior sum to $360°$.....

$$\frac{36° \times n}{36°} = \frac{360°}{36°}$$

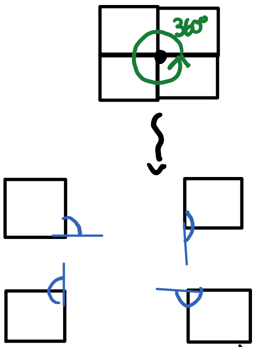
$n = 10$

Answer: 10 sides

exercise: A soccer ball is made with regular pentagons and hexagons as shown. could the same shapes and pattern be used to tile a floor? Explain.



Normal Tiles:
Squares



Each internal angle is 90°, ∴ each external is 90°

For tiles to fit together with no gaps, The shapes' exterior angles which form a point, must add to 360°.

Soccer Ball Pattern.

At any point, the exterior angles of 2 hexagons and 1 pentagon come together.

180° − interior = exterior.

Hexagon: $180° - 120° = 60°$

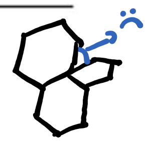
Pentagon: $180° - 108° = 72°$

$2(H) + (P) \stackrel{?}{=} 360$

$2(60°) + (72°) \stackrel{?}{=} 360$

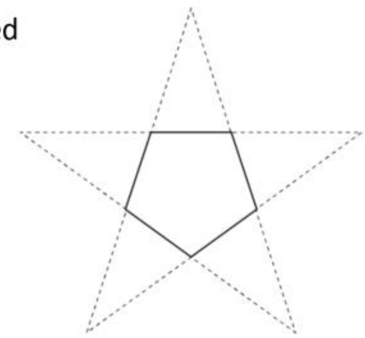
$120° + 72° \stackrel{?}{=} 360°$

$192° \neq 360°$ ⇒



Answer: No, we would have a 168° gap in the tiles.

exercise: The sides of a regular pentagon are extended to make a 5-pointed star. What is the angle at each point of the star?



Answer: ______