4.3 - Graphs of y=a(x-h)^2+k

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As we've seen, quadratic functions can be represented in the form:

$$f(x) = ax^2 + bx + c \text{ , } a \neq 0$$

This is known as **general form**.

Consider the functions $f(x) = -3(x + 5)^2 – 2$ and $g(x) = -3x^2 – 30x – 77$.

Clearly, g(x) is in general form, but try expanding and simplifying f(x):

$$(x+5)(x+5) = x^2+5x+5x+25$$
$$= x^2+10x+25$$

$$f(x) = -3(x+5)^2-2$$
$$f(x) = -3(x^2+10x+25)-2$$
$$f(x) = -3x^2-30x-75-2$$
$$f(x) = -3x^2-30x-77$$

They're the same!

The above function f(x) was given in what's known as **standard form**:

$$f(x) = a(x – h)^2 + k \text{ , } a \neq 0$$

Standard form is also known as **vertex form** because writing a quadratic function in this way makes determining the vertex very simple.

The coordinates of the vertex given in standard form are always (-*h*, *k*)

Note: The negative sign in front of the *h* value means to multiply it by -1.

Example: Determine the vertex of the following quadratic functions.

$y = 2(x + 4)^2 – 4$

$h = 4$, $k = -4$ } vertex:
$(-h, k)$
$(-4, -4)$

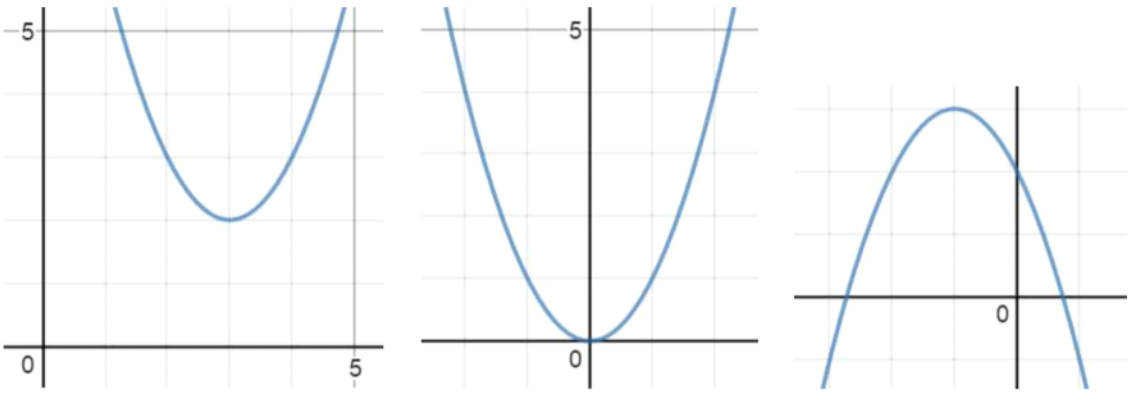
$f(x) = -6(x – 3)^2 + 1$

$h = -3$, $k = 1$ } vertex:
$(-h, k)$
$(-(-3), 1)$
$(3, 1)$

As for the "a" value, it has the same properties as the "a" value from the general form.

That is, if $a < 0$, the parabola opens downward
$a > 0$, the parabola opens upward

Depending on where the vertex of a parabola is, and whether it opens up or down, the graph can have 0, 1, or 2 x-intercepts:

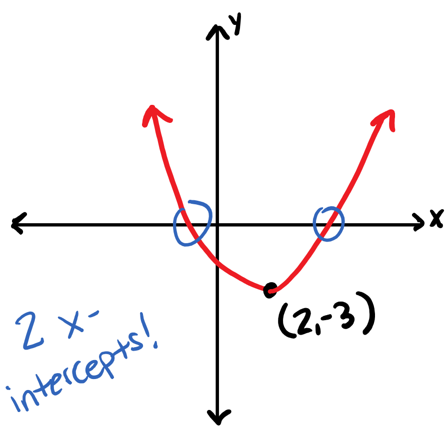


Example: Determining the number of x-intercepts given an equation.

How many x-intercepts do the following parabolas have?

$y = 4(x - 2)^2 - 3$ ↳ vertex: (2,-3)
↳ opens: upwards

Sketch:



$y = -(x + 4)^2 + 0$ ↳ vertex: (-4,0)
↳ a = -1
↳ opens: down