4.4 - Graphs of y=a(x-m)(x-n)

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In this section, we explore a third a final form of quadratic functions:

$$f(x) = a\underline{(x - m)}\underline{(x - n)}\ ,\ a \neq 0$$

(or in equation form): $y = a(x - m)(x - n)\ ,\ a \neq 0$

This is known as **factored form** because it is general form factored in to 2 binomial factors.

Because the 2 factors are multiplied together, if one binomial is 0, the entire equation will become 0, that is y = 0, which we know as the x-intercept(s).

Example: Determining the x-intercept(s) in a factored form quadratic.

$f(x) = (x – 3)(x + 2)$

What values of x will make the entire equation equal 0?

$(x-3)(x+2)$

if $x=3$, then...
$(x-3)$
$=(3-3)$
$=0$

if $x=-2$, then...
$(x+2)$
$(-2+2)$
$=0$

$\therefore$ the x-intercepts for $f(x)=(x-3)(x+2)$ are 3 and -2

$\Rightarrow f(3)=(3-3)(3+2)=(0)(5)=0$, $\Rightarrow f(-2)=(-2-3)(-2+2)=(-5)(0)=0$

Given that y-intercepts occur when x = 0 (and vice-versa for x-intercepts), if we substitute in 0's for our x's, we can find a pattern to find the y-intercept:

$f(x)=(x-3)(x+2)$
$f(0)=(0-3)(0+2)$
$f(0)=(-3)(2)=-6$

$\therefore$ The y-intercept is -6

We would also notice that if we graphed several functions, the equation of the axis of symmetry would be:

$$x = \frac{m+n}{2}$$

Example: Determining the axis of symmetry.

y = 4(x – 4)(x – 2)

$$x = \frac{m+n}{2} = \frac{(4)+(2)}{2} = \frac{6}{2} = 3$$

m=4
n=2 } x-int's

∴ The axis of symmetry is x=3

As always, our "a" value tells us which way the parabola opens:

That is, if a < 0 , the parabola opens downward
a > 0 , the parabola opens upward

The disguised "a" value:

Graph the function y = -2(3 – x)(x – 7) on desmos. What do you notice?

The graph opens upward..?

If a function is not *exactly* in the form f(x) = a(x – *m*)(x – *n*), you'll need to rearrange it to accurately predict it's properties.

$f(x) = -2(3-x)(x-7)$

↳ backwards

Let's correct the above equation:

y = -2(3 – x)(x – 7)

$y = -2(-x+3)(x-7)$

→ can't have a negative x value. Factor out a -1 to make it positive.

combine

$y = \underbrace{(-2)(-1)}(x-3)(x-7)$

$y = 2(x-3)(x-7)$

↳ now, our a-value is positive, and the equation is fully in factored form. This explains why the graph opens up.

Example: Determine the equation of a graph.

A quadratic equation passes through the points (-8 , 0), (2 , 0), and (0 , 48)

y = 0, ∴ They are x-ints.

x = 0, ∴ It is a y-int.

Is this information useful in:

✗ ① General Form (ax^2+bx+c)

✗ ② Standard Form ($a(x-h)^2+k$)

✓ ③ Factored Form ($a(x-m)(x-n)$) ?

↳ m and n are x-ints, so we can substitute them in to factored form:

$y = a(x-m)(x-n)$

$y = a(x+8)(x-2)$

Still need the a-value...

• Because we know the point (0, 48) [x, y] lies on the parabola, we can substitute x = 0 and y = 48 in to the equation and solve for a:

$y = a(x+8)(x-2)$

$(48) = a(0+8)(0-2)$

$48 = a(8)(-2)$

$48 = -16a$

$\div 16 \quad \div 16$

$a = 48/-16 = -3$

Now substitute a = -3 for our final equation:

$\boxed{y = -3(x+8)(x-2)}$