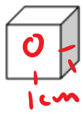
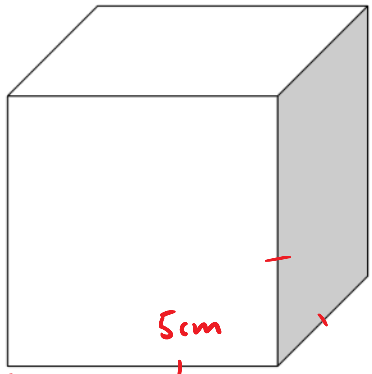
FOUNDATIONS OF MATH 11 Ch. 1 – Day 5: Linear Scale Factors, Surface Area, and Volume

SCALE FACTORS FOR SURFACE AREA

Compare the surface areas of a 1-cm cube and a 5-cm cube.





The linear scale factor,

$k_{linear} = \frac{\text{Diagram length}}{\text{Original length}} = \frac{5cm}{1cm} = 5$

The surface area scale factor is

$SA = 6lw$ (1cm) $= 6(1cm)(1cm) = 6cm^2$

$SA = 6lw$ (5cm) $= 6(5cm)(5cm) = 150cm^2$

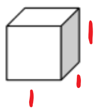
$k_{surface\ area} = \frac{\text{Diagram Surface Area}}{\text{Original "}} = \frac{150}{6} = 25$

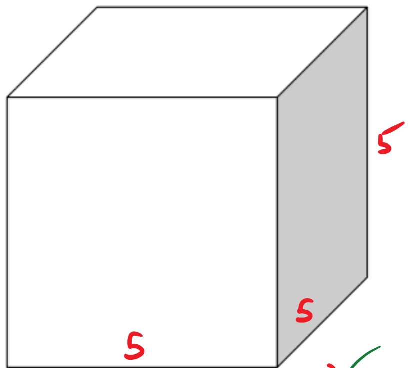
The ***surface area scale factor*** is $\frac{\text{model surface area}}{\text{actual surface area}}$ $k_{area} = k_{surface\ area}$

which is equal to k^2_{linear}, the square of the linear scale factor.

SCALE FACTORS FOR VOLUME

Compare the volume of a 1-cm cube and a 5-cm cube.





The linear scale factor,

From above, $k_{linear} = 5$

$V_{cube} = lwh$

The volume scale factor is

$V = (1cm)(1cm)(1cm) = 1cm^3$ (1cm)

$V = (5cm)(5cm)(5cm) = 125cm^3$ (5cm)

$k_{volume} = \frac{\text{Diagram Volume}}{\text{Original Volume}} = \frac{125cm^3}{1cm^3} = 125$

The ***volume scale factor*** is $\frac{\text{model volume}}{\text{actual volume}}$ ie. $k_{volume} = (k_{linear})^3$

which is equal to k^3_{linear}, the cube of the linear scale factor.

example: A 1:50 die-cast model of a dump truck is shown. The model can carry 1200 cm³ of sand. How much sand will the actual dump truck be able to carry?

K_{linear}

$1:50 = \frac{1}{50} = 0.02$

- linear scale factor,

$K_{linear} = 0.02$,

Sand is being measured in volume, so we must use K_{volume}.

$K_{volume} = [K_{linear}]^3 = [0.02]^3 = 0.000008$

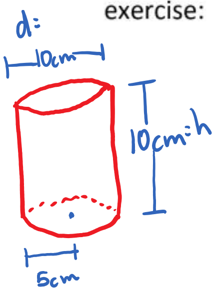
- Find the truck load capacity,

$K_{volume} = \frac{\text{Diagram Volume}}{\text{Original Volume}}$

$[0.000008] = \frac{[1200\,cm^3]}{x}$ ⇒ $x = \frac{[1200\,cm^3]}{[0.000008]} = 150{,}000{,}000\,cm^3$

$= 150\,m^3$

exercise: A cylindrical soup can is 10 cm wide and 10 cm tall. The company wants to make a similar shaped can that will hold twice as much soup. What will the diameter and height of the new can be?



- volume scale factor of the can

$K_{volume} = 2$

- linear scale factor of the can

$\sqrt[3]{K_{volume}} = \sqrt[3]{(K_{linear})^3}$

$K_{linear} = \sqrt[3]{K_{volume}} = \sqrt[3]{2} \cong 1.25$

- diameter and height of the can,

Diameter: $K_{linear} = \frac{\text{Diagram Length}}{\text{Original Length}}$ ⇒ $(1.25) = \frac{x}{(10cm)}$ ⇒ $x = (1.25)(10cm)$ $= 12.5cm$

Height: $K_{linear} = \frac{\text{Diagram}}{\text{Original}}$ ⇒ $(1.25) = \frac{x}{(10cm)}$ ⇒ $x = (1.25)(10cm)$ $= 12.5cm$

So, in general, we need to remember two things:

Firstly, what quantities are considered to be "linear," "area," etc… so we can use the right one

Quantity	Dimension	Scale Factor to Use
Any straight distance, perimeter, height, length, width, radius	Linear (1-D)	k_{linear}
Area, surface area	Area (2-D)	k_{area}
Volume	Volume (3-D)	k_{volume}

Secondly, if we're given one scale factor but we need another to complete the problem, we need to know how to convert between the three we know. This is known as the square-cube law: