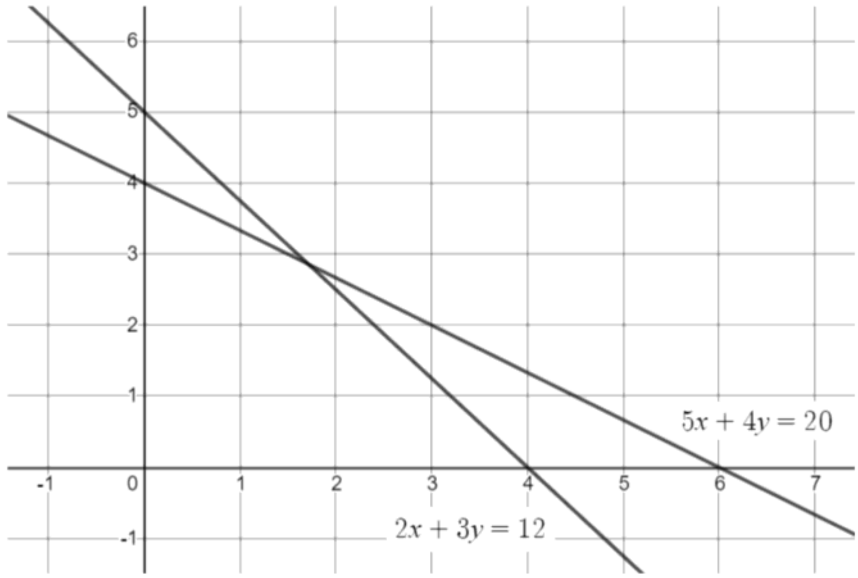
5.5 - Determining an Optimal Solution

January 22, 2020 10:40 AM

Recall the problem from the last section with Janine's clothing store where she was selling sweaters and vests. The solution to what combination of sweaters and vests she could make looked like this:



The next logical question for Janine to ask goes from what <u>can</u> she make to what <u>should</u> she make? In this case we're talking about money. More specifically, we want to know what combination of sweaters and vests Janine can make to give her the most profit.

A problem like this is an "optimization" problem because we're trying to optimize something. In this case, we want the <u>most</u> of something. Other times, you will want the <u>least</u> of something. It all depends on the context of the question.

For now, lets solve Janine's problem. Let's say Janine makes a profit of \$40 for every sweater she sells, and \$50 for every vest she sells. We'll need to complete the following table to see what combination is most profitable from the combinations we've already discovered are possible. The formula 40x + 50y will give us profit:

Sweaters

Vests

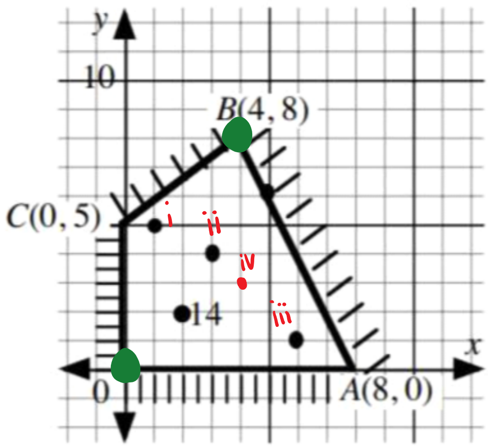
x	0	0	0	0	0	1	1	1	1	2	2	2	3	3	4
y	0	1	2	3	4	0	1	2	3	0	1	2	0	1	0
40x+50y	0	50	100	150	200	40	90	140	190	80	130	180	120	170	160

most profitable!

Therefore, the optimized solution for how many vests and sweaters Janine should make for maximum profit is:

0 sweaters
4 vests.

The region *OABC* represents the solution region of a system of linear inequalities.



On one of the given points in the solution area (2,2), the value of 2x + 5y is calculated to be 14.

Determine the value of 2x + 5y at the other marked points in the solution area, then calculate the values for all the whole number points.

i (1,5)
$2x+5y$
$2(1)+5(5)$
$=27$

ii (3,4)
$2x+5y$
$2(3)+5(4)$
$=26$

iii (6,1)
$2x+5y$
$2(6)+5(1)$
$=17$

iv (4,3)
$2(4)+5(3)$
$=23$
...

A (8,0)
$2(8)+5(0)$
$=16$

B (4,8)
$2(4)+5(8)$
$=48$
max!

C (0,5)
$2(0)+5(5)$
$=25$

O (0,0)
$=2(0)+5(0)$
$=0$ min!

In the solution region, determine the maximum value of 2x + 5y

(4,8) ⇝ 48

In the solution region, determine the minimum value of 2x + 5y

(0,0) ⇝ 0

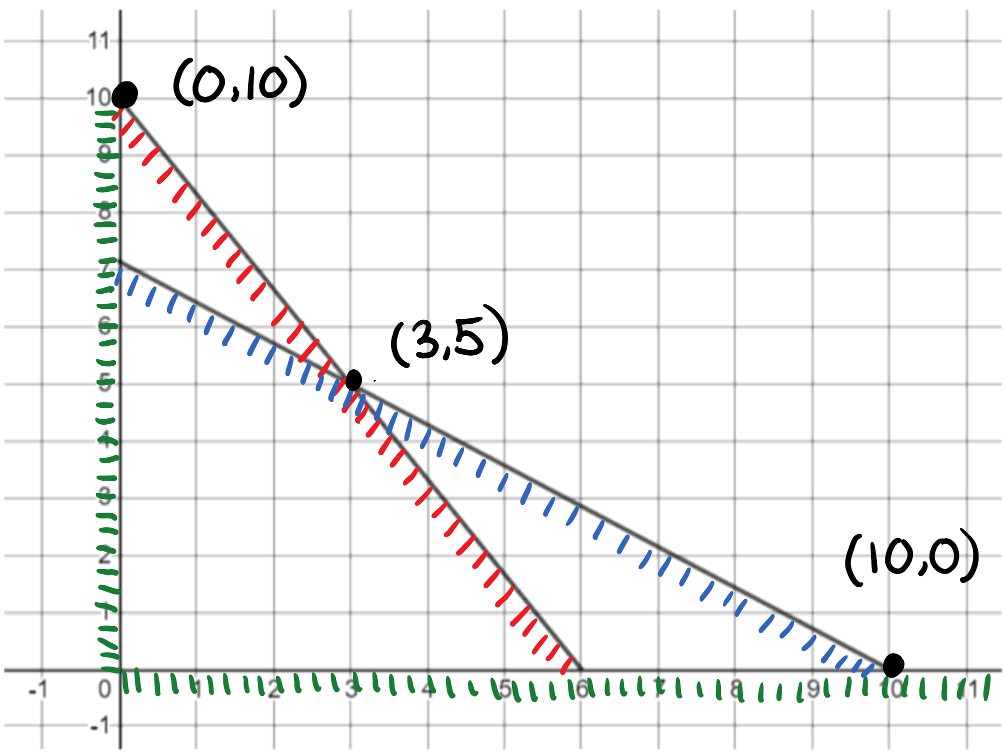
Important:

In general, if x and y are natural or whole numbers, the optimal solution will be found on or nearest to a boundary of the solution region.

If, however, x and y are real numbers (whole numbers and all numbers in between) the optimal solution will always be found at one of the corners of the solution region.

Example

The diagram below shows two inequalities. The shaded region for both lies above them. Also, the shaded region should be restricted to quadrant I (positive values for x and y only). Use fringe shading to represent the solution region.



By using each vertex of the solution area, determine the minimum value of:

i) $4x + 5y$

$(0,10)$: $4(0)+5(10)$ $0+50$ $=50$

$(3,5)$: $4(3)+5(5)$ $12+25$ $=37$ ☆

$(10,0)$: $4(10)+5(0)$ $40+0$ $=40$

min occurs at $(3,5)$ for $4x+5y$

ii) $2x + y$

$(0,10)$: $2(0)+(10)$ $=0+10$ $=10$ ☆

$(3,5)$: $2(3)+(5)$ $=6+5$ $=11$

$(10,0)$: $2(10)+(0)$ $=20+0$ $=20$

min at $(0,10)$ for $2x+y$.

iii) $x + 4y$

$(0,10)$: $(0)+4(10)$ $=0+40$ $=40$

$(3,5)$: $(3)+4(5)$ $=3+20$ $=23$

$(10,0)$: $(10)+4(0)$ $=10+0$ $=10$ ☆

min at $(10,0)$ for $x+4y$