5.6 - Modelling & Linear Programming

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FOUNDATIONS OF MATH 11 Ch. 5 – Day 6: Modelling & Linear Programming

So far you've been given the equations and inequations to use, now you'll have to create them on your own.

You'll want to keep an eye out for key words when doing so:

Phrase	Symbol
Fewer than, less than	<
Up to, more than, a maximum of, not greater than, less than or equal to	≤
More than, greater than	>
At least, a minimum of, greater than or equal to	≥

Example

Determine two inequalities in two variables for each statement.

a) The sum of two whole numbers is less than 10.

let x = first #

let y = second #

$x + y < 10$

$x < 10 - y$

b) The difference between the ages of two girls is at least 4.

let x = age of older girl

let y = " " younger "

$x - y \geq 4$

$+y \quad +y$

$x \geq 4 + y$

c) A gardener uses up to 30 square meters to plant peas and carrots.

let x = area of peas

let y = area of carrots

$x + y \leq 30$

$x \leq 30 - y$

Recall the sweater and vest problem from the previous two sections. We followed the following procedure to arrive at a solution:

1. Define variables for the unknown quantities in the question.

let x = # of sweaters
let y = # of vests

2. Form inequalities with those variables from the information in the problem.

$5x + 4y \leq 40$

$2x + 3y \leq 5$

3. Graph the inequalities to determine the solution region.

See previous section...

4. Determine the function to be maximized or minimized. This is called the "**objective function**".

Profit = $40x + 50y$

5. Determine the optimal value of the objective function in the solution region.

(0,4) – no sweaters
– 4 vests.

6. Answer the problem.

Given that she had 2000g of wool and 36 hours, in order to make the most profit, she should make 4 vests.

Example

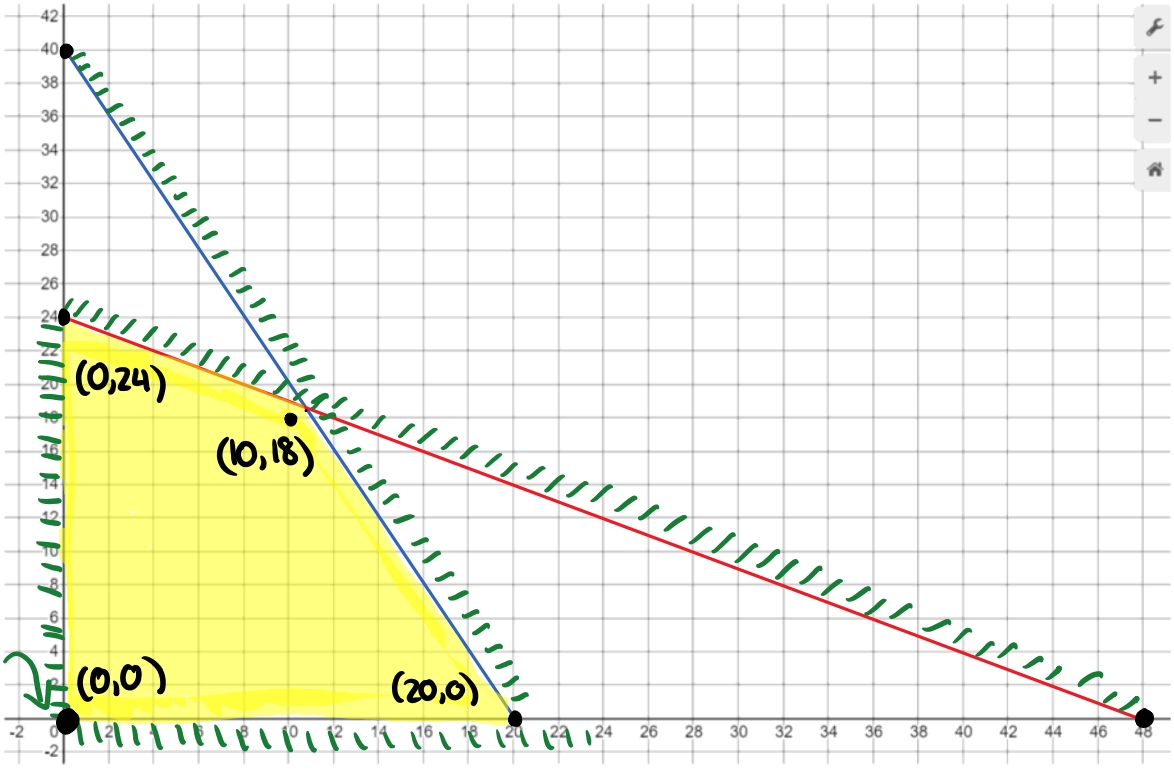
Use the above method to answer the following question:

One kind of cake requires 200g of flour and 25g of butter (cake "A") and another kind of cake requires 100g of flour and 50g of butter (cake "B"). Suppose we want to make as many cakes as possible but we only have 4kg (4000g) of flour and 1.2kg (1200g) of butter. Assuming no other ingredient will limit us, how many cakes of each kind should we make?

let x = # of cake A
let y = # of cake B

$\begin{cases} 200x + 100y \le 4000 \\ 25x + 50y \le 1200 \end{cases}$ reduce → $\begin{cases} 2x + y \le 40 \\ x + 2y \le 48 \end{cases}$

Also, $x \ge 0$, $y \ge 0$



Graph using intercepts:

$2x + y = 40$

x-int: $2x + (0) = 40$, $x = 20$

y-int: $2(0) + y = 40$, $y = 40$

$x + 2y \le 48$

x-int: $x + 2(0) = 48$, $x = 48$

y-int: $(0) + 2y = 48$, $y = 24$

Test: $2(0) + (0) \le 40$

$0 \le 40$

✓ True

Shade there

$(0) + 2(0) \le 48$

$0 \le 48$

✓ True!

Objective Function:

$x + y$ (total cakes)

Check: (0,0) ; $x + y = 0 + 0 = 0$ ☹

(20,0) ; $x + y = 20 + 0 = 20$

(0,24) ; $x + y = 0 + 24 = 24$

(10,18) ; $x + y = 10 + 18 = 28$ ☺ ★

∴ we should make
10 A cakes
18 B cakes