4.1 - Review & Preview

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Unit 4: Exponential & Logarithmic Functions

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Recall powers:

Power 2^5 Exponent

Base

Also recall the following exponent laws:

The Zero Exponent: $x^0 = 1$

The Product Law: $x^m x^n = x^{m+n}$

The Quotient Law: $x^m \div x^n = x^{m-n}$

Power of a Power: $(x^m)^n = x^{m \cdot n}$

Power of a Product: $(xy)^m = x^m y^m$

Power of a Quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

Integral Exponent Rule: $x^{-m} = \frac{1}{x^m}$

Rational Exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Use the exponent laws to simplify the following.

a) $x^2 \times x^3$
$= x^{2+3}$
$= x^5$

b) $\frac{x^6}{x^2}$
$= x^{6-2}$
$= x^4$

c) $(x^5)^4$
$= x^{5 \cdot 4}$
$= x^{20}$

d) $\left(\frac{x}{y}\right)^3$
$= \frac{x^3}{y^3}$

e) $(3x)^3$
$= 3^3 x^3$
$= 27x^3$

Write each expression without brackets and with positive exponents.

a) x^{-3}

$= \frac{1}{x^3}$

b) $(4x^3)(2x^{-4})$ multiplied, so add

$= (4)(2)x^3x^{-4}$

$= 8x^{-1} = 8/x$

c) $(2x^4)^3$

$= 2^3(x^4)^3$

$= 8x^{4\cdot3} = 8x^{12}$

d) $5(3x^2)^3$

$= 5(3^3(x^2)^3)$

$= 5(27x^{2\cdot3})$

$= 5(27x^6)$

$= 135x^6$

e) $\frac{5x^{-3}}{x^{-2}}$

$= 5\frac{x^{-3}}{x^{-2}}$

$= 5x^{-3-(-2)}$

$= 5x^{-1} = 5/x$

f) $\frac{12b^{-\frac{1}{2}}}{3b}$

$= 4\frac{b^{-1/2}}{b^1}$

$= 4b^{-1/2-1} = 4b^{-3/2}$

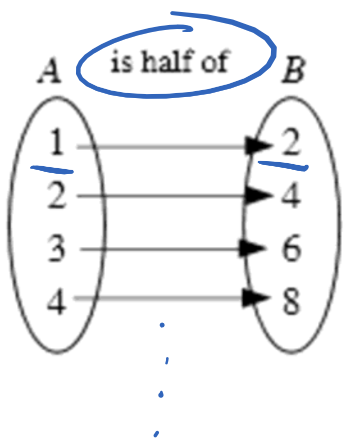
$= \frac{4}{b^{3/2}} = \frac{4}{\sqrt[2]{b^3}}$

Now we're going to review **relations** (and a special relation called a **function**) and introduce the **inverse** of a relation.

Put simply, a function or relation tells us how to get from one number to another number.

For example, to get from 3 to 6, we could say the function is, "add three." We may also say the function is, "double the first number."

Typically, however, we do this with a **set** of numbers:



As you can see, set A contains 4 elements, and each one is mapped to a new element in set B. The function in this case is, “half of.”

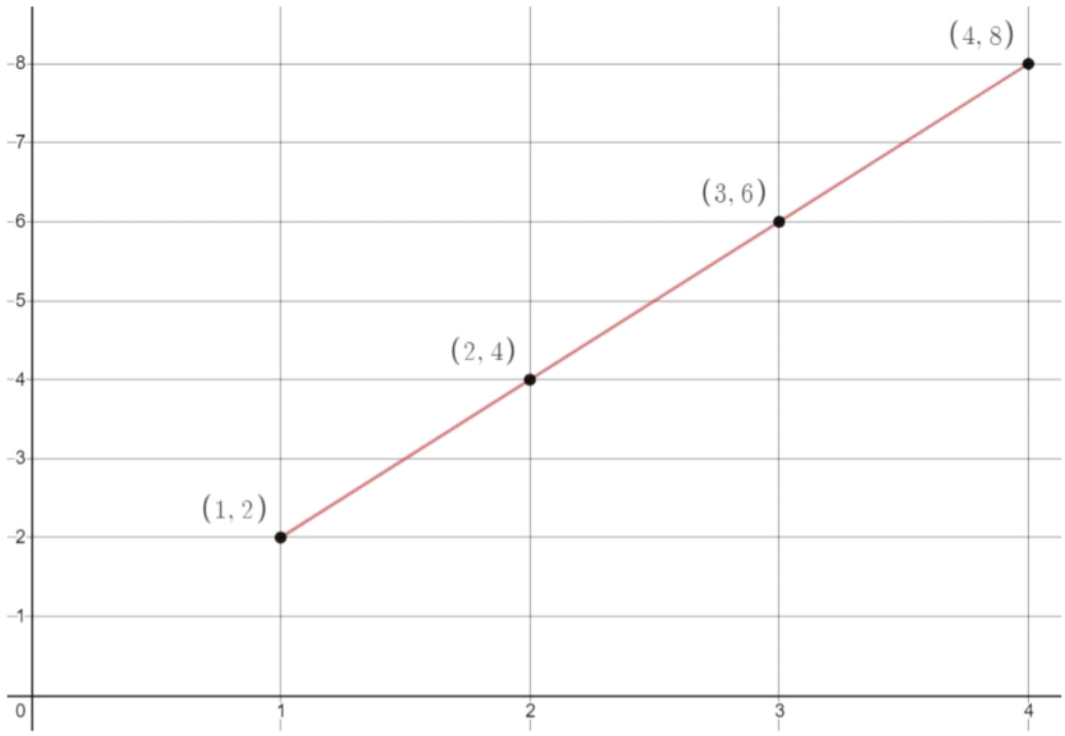
The set where we start (set A) is called the **domain**. In this case, $A = \{1, 2, 3, 4\}$
The set where we end (set B) is called the **range**. In this case, $B = \{2, 4, 6, 8\}$

Since we have two sets, we can graph this relation on a graph which has two axes.

We can create the points to graph by assigning set *A* to the x-axis, and assigning set *B* to the y-axis. Let’s list them below:

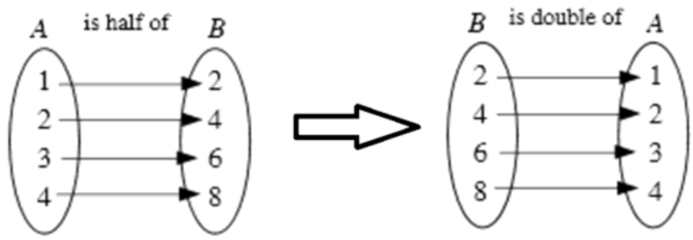
$$\{(1,2), (2,4), (3,6), (4,8)\}$$

We should know how to graph this already, but for completeness sake, I’ve placed it below:



Now let's discuss the **inverse** of a function or relation. The inverse sort of "undoes" what the original function did.

For example, the inverse of the previous relation, "is half of," would be, "is double of." One way to do this is instead of mapping elements from set *A* to elements of set *B*, we can map them from set *B* to set *A*:

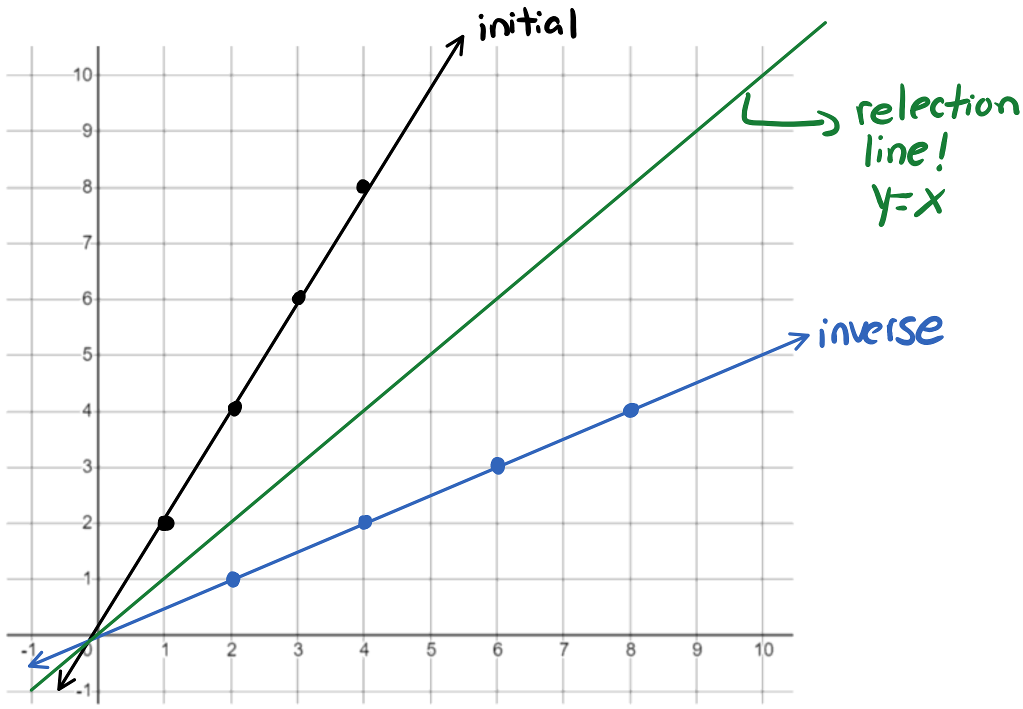


This of course also means that the domain and range have been swapped.

We can also graph the inverse relation. This time, set *B* will be plotted on the x-axis, and set *A* will be plotted on the y-axis. Let's list the coordinates now:

$$\{(2,1),(4,2),(6,3),(8,4)\}$$

Now we have one relation and its inverse. Let's plot both relations on the same graph:



Notice how both relations seem to be mirrored from each other. Let's go back and draw a dotted line that represents this mirror (mathematically known as a "**reflection**").

The equation of this mirror line (also known as an "**axis of symmetry**") will always be:

$$y = x$$

Let's summarize our findings:

	Function in Words	Coordinates of Points on the Graph	Equation	Domain	Range
Function	is half of	$(1,2)$, $(2,4)$, $(3,6)$, $(4,8)$	$y = 2x$	$\{1,2,3,4\}$	$\{2,4,6,8\}$
Inverse	is double of	$(2,1)$, $(4,2)$, $(6,3)$, $(8,4)$		$\{2,4,6,8\}$	$\{1,2,3,4\}$

Notice how the x and y coordinates for the points were swapped to get the inverse. If we want to find the **equation** of the inverse, we do the same: swap the x and y coordinates, and isolate for the new "y" variable:

$$y = 2x \rightsquigarrow \frac{x}{2} = \frac{\cancel{2}y}{\cancel{2}} \qquad y = \frac{1}{2}x = \frac{x}{2}$$

Now let's do an identical exercise, but this time with a slightly more complicated relation:

The first diagram shows the relation "is the square root of" on the domain $\{-3, -2, 2, 3\}$.

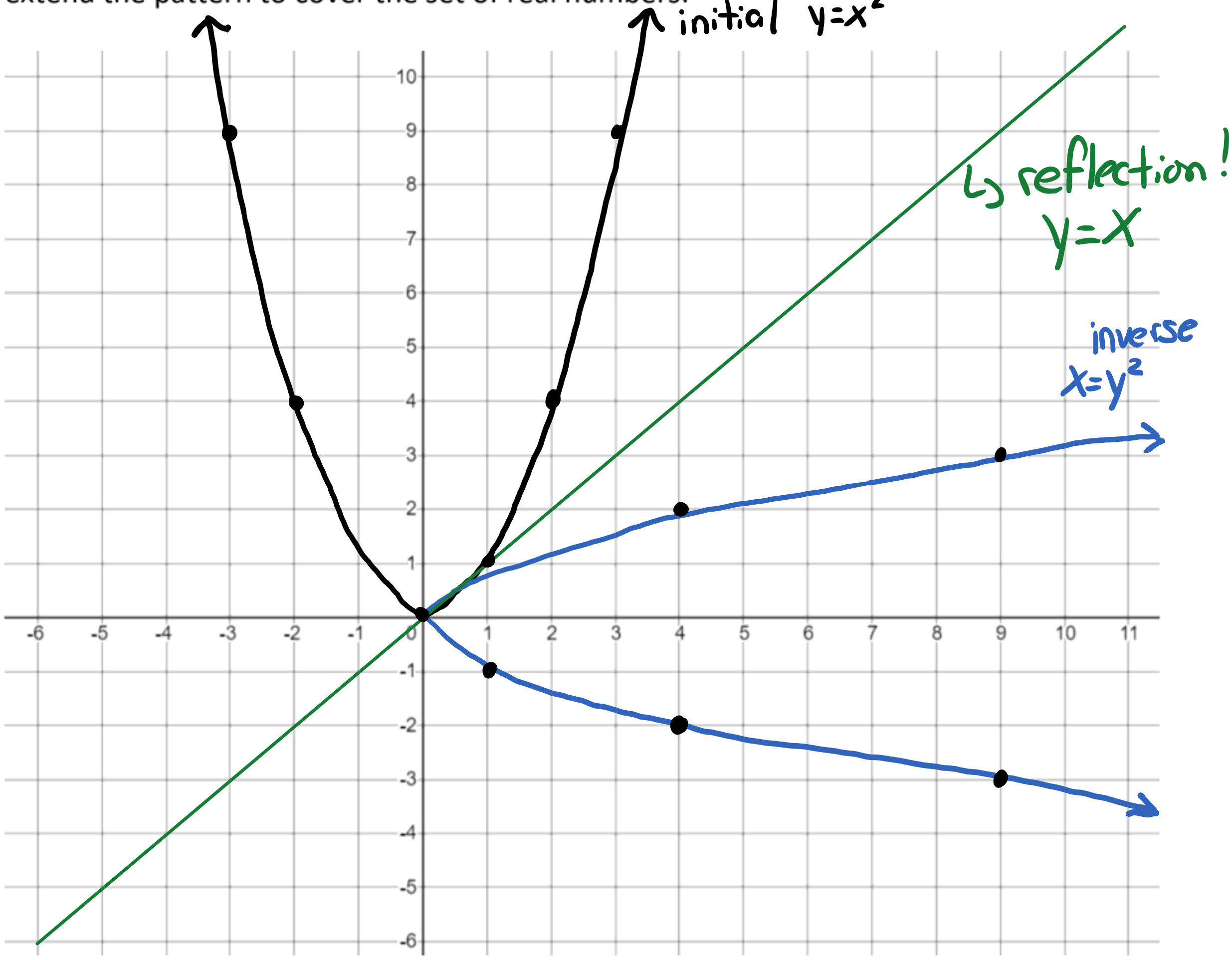
What would be the inverse of, "is the square root of," be? Fill in the above diagram to get the inverse.

"is the square of"

Let's now fill in the following chart so we can get a good idea of what's happening:

	Relation in Words	Coordinates of Points on the Graph	Equation	Domain	Range
Relation	is the square root of	$\{(-2,4),(2,4),(-3,9),(3,9)\}$	$y=x^2$	$\{\pm 2, \pm 3\}$	$\{4,9\}$
Inverse	is the square of	$\{(4,-2),(4,2),(9,-3),(9,3)\}$	$x=y^2$	$\{4,9\}$	$\{\pm 2, \pm 3\}$

Now let's plot the points of both relations on the graph below, join them with a curve, and extend the pattern to cover the set of real numbers:



Again, notice how both relations are reflected along the line $y = x$.

When discussing inverse relations, the following will always be true:

- The inverse of a relation is defined by swapping the x and y – coordinates.
- If the relation is given in equation form, the inverse of the relation can be obtained by:
  1. Interchanging x and y
  2. Isolating for y
- If the relation is given to you graphically, the inverse can be obtained by reflecting the graph along the line $y = x$.
- When taking the inverse of a relation, the old domain becomes the new range, and the old range becomes the new domain.

Example

Determine the inverse of the function defined by the equation $y = 5x - 6$.

Swap x and y:

$y = 5x - 6 \rightsquigarrow x = 5y - 6$

Isolate for y:

$x = 5y - \cancel{6}$

$+6 \qquad \cancel{+6}$

$\frac{x+6}{5} = \frac{\cancel{5}y}{\cancel{5}}$

$\rightarrow y = \frac{x+6}{5}$

$y = \frac{1}{5}x + \frac{6}{5}$