4.2 - Analyzing Exponential Functions

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An exponential function is a function in the form:

$$f(x) = ab^x$$

$y = ab^x$

where, $a \neq 0\,, b > 0\,, and\ b \neq 1$

We're going to begin our discussion of exponential functions by comparing two different ones: $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$. We'll start by graphing them using a table of values:

$y = 2^x$

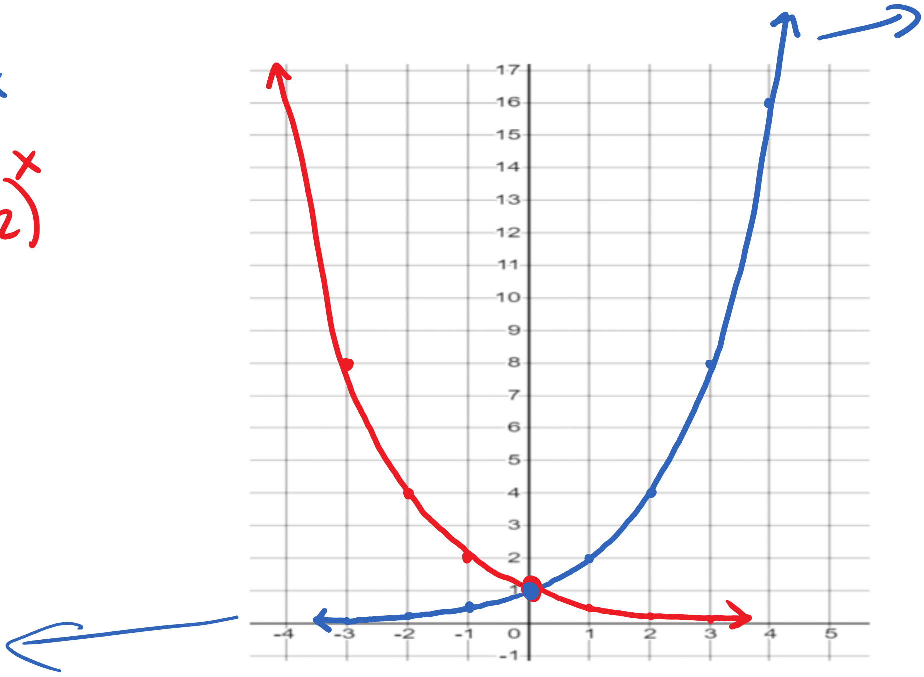
$y = 2^{(-3)} = \frac{1}{2^3} = \frac{1}{8}$

$y = 2^x$	
x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16

$y = (1/2)^x$	
x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8
4	1/16

$y = 2^x$

$y = \left(\frac{1}{2}\right)^x$



Notice how the graph gets ***very*** close to the x-axis, but never truly touches it.

We call this an "asymptote"

Let's complete the following table to keep track of what's happening:

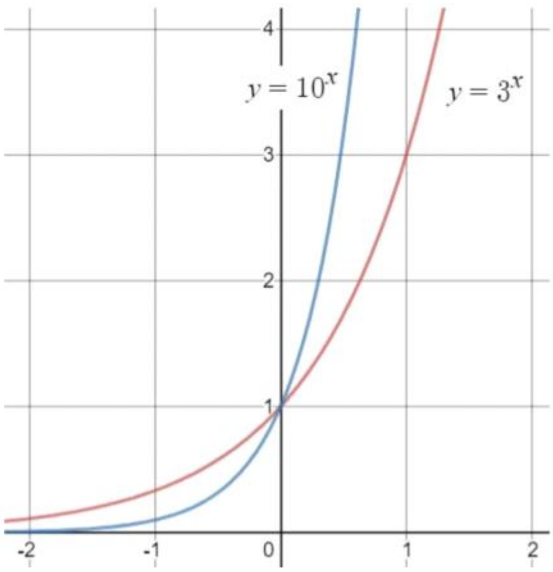
Equation of Function	*Domain of Function*	*Range of Function*	*x-intercept of Graph*	*y-intercept of Graph*	*Equation of Asymptote*
$y = 2^x$	$x \in \mathbb{R}$	$\{y \mid y > 0, y \in \mathbb{R}\}$	none	1	$y = 0$
$y = \left(\frac{1}{2}\right)^x$	$x \in \mathbb{R}$	$\{y \mid y > 0, y \in \mathbb{R}\}$	none	1	$y = 0$

"Reading" the above graphs from left to right, which one is increasing and which one is decreasing? $y = 2^x$ increase, $y = (1/2)^x$ decrease

Also, the graphs are reflections in the y – axis.

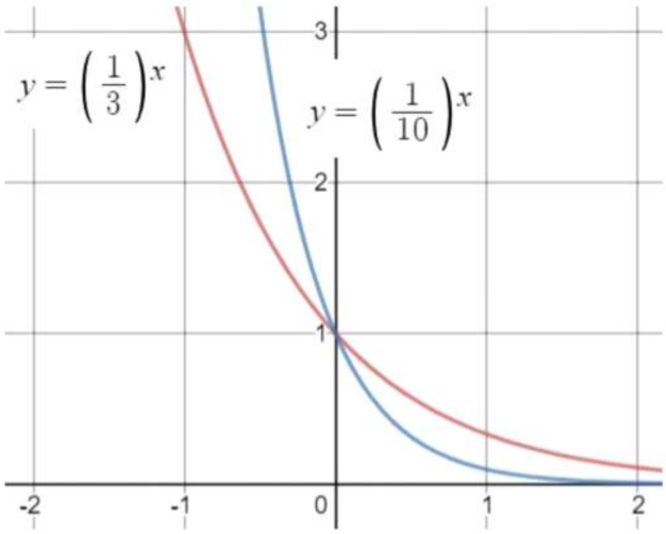
Exploring the *b*-value in $y = ab^x$, when $a = 1$

Just like the graphs we've seen so far, changing the values in the equation will affect the shape of the graph:



As we can see, as the *b*-value increases, the steepness of the graph increases as well.

However, this is only true when the *b*-value is equal to or larger than 1 .



Here I've graphed two more functions whose *b*-values are between 0 and 1.

Although $^1/_3$ is technically larger than $^1/_{10}$, that graph is less steep.

Therefore the rule is backwards when *b* is between 0 and 1.

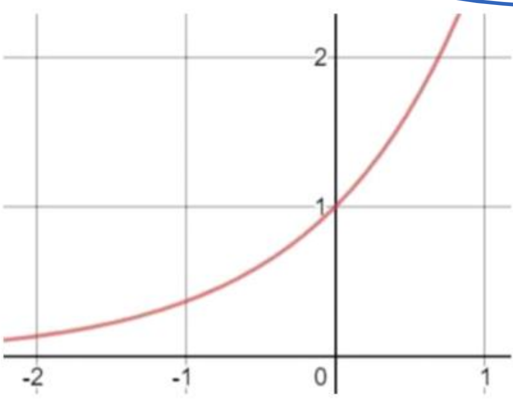
Remember the number π? It's an important number that pops up in many fields of science. It's an irrational number (which means it can't be expressed as a fraction) we get from dividing the circumference of a circle by its diameter.

There's a similar irrational number that exists called " *e* ", also known as "Euler's constant"

e is approximately equal to 2.718 (it's an irrational number, so it continues forever just like π).

We get it from the following pattern:

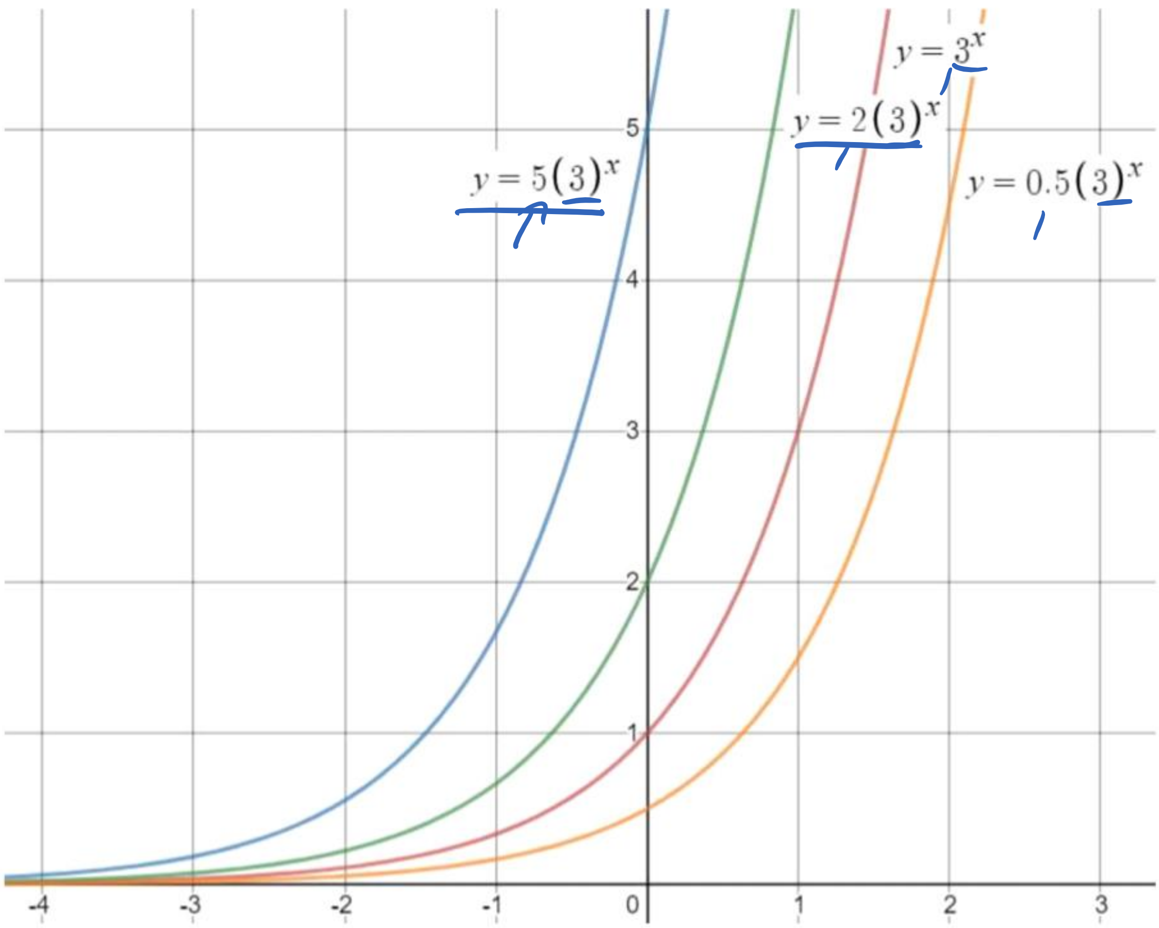
$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$



We're not going to discuss the number *e* at length in this course, just understand that it also appears very often in many fields of science, and particularly the function $y = e^x$ has some very interesting properties. Because *e* ≈ 2.718, you can imagine the graph $y = e^x$ would have a shape between $y = 2^x$ and $y = 3^x$. I've graphed it here for the sake of completeness.

Exploring the *a*-value in $y = ab^x$, when $a > 1$

Now let's see what happens when we lock the *b*-value at 3, and alter the *a*-value:

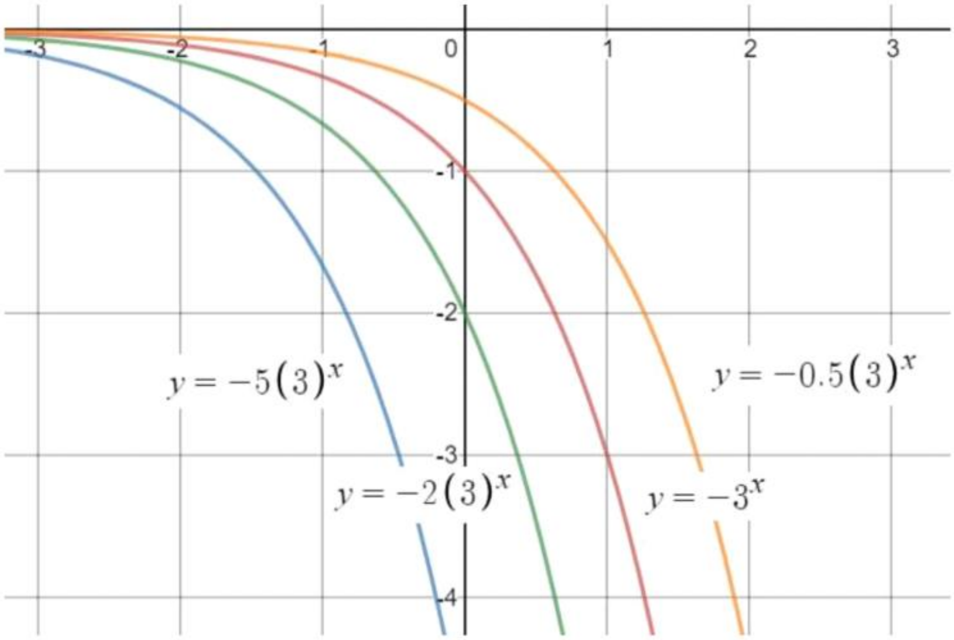


What do you notice about the *a*-value and the y-intercept of the graph?

In exponential functions, $y=ab^x$, a will be the y-int.

ie: $(0, a)$

What about if the *a*-value is negative?



As we can see, the *a*-value still corresponds to the graph's y-intercept.

	$y=ab^x$ where $a>0$	$y=ab^x$ where $a<0$
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$\{y \mid y>0, y \in \mathbb{R}\}$	$\{y \mid y<0, y \in \mathbb{R}\}$
x-intercept	—	—
y-intercept	a	a
Asymptote	$y=0$	$y=0$

Finally, here is a section which details the patterns we've explored so far:

The following summarizes the basic characteristics of the graph of the exponential function with equation $y = ab^x$, where $a \neq 0, b > 0, b \neq 1$.

Use the information from the previous explorations to complete the following.

- The y-intercept is __a__.
- There is __no__ x-intercept.
- The x-axis is a __horizontal__ __asymptote__.
- The domain is __$x \in \mathbb{R}$__.
- The range depends on the value of a.
  - For $a > 0$, the range is __$\{y \mid y > 0, y \in \mathbb{R}\}$__.
  - For $a < 0$, the range is __$\{y \mid y < 0, y \in \mathbb{R}\}$__.

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- The values of a and b determine whether the graph is increasing or decreasing.
  - When $a > 0$ and $b > 1$, the graph __$\uparrow$__ from quadrant 2 to quadrant 1.
  - When $a > 0$ and $0 < b < 1$, the graph __$\downarrow$__ from quadrant 2 to quadrant 1.
  - When $a < 0$ and $b > 1$, the graph __$\downarrow$__ from quadrant 3 to quadrant 4.
  - When $a < 0$ and $0 < b < 1$, the graph __$\uparrow$__ from quadrant 3 to quadrant 4.
- The value of b also affects the steepness of the graph as x increases.
  - When $b > 1$, the curve gets steeper as b __increases__.
  - When $0 < b < 1$, the curve gets steeper as b __decreases__.