4.3 - Analyzing Logarithmic Functions

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Recall from section 1 of this unit the concept of an **inverse function**. We get the inverse of a function when we interchange the x and y variables, and isolate for y.

Example

Determine the inverse of the function $y = 4 - 2x$

Switch x & y: $y = 4 - 2x \rightsquigarrow x = 4 - 2y$

-isolate for y:-

$x - 4 = -2y$

$y = -\frac{1}{2}x + 2$

We're now going to explore the inverse of the exponential functions we explored in the last section.

Let's determine the inverse of the exponential function $y = 2^x$:

$y = 2^x \rightsquigarrow x = 2^y$

?

We're stuck here. We don't know the math required to isolate for "y" after we interchange them.

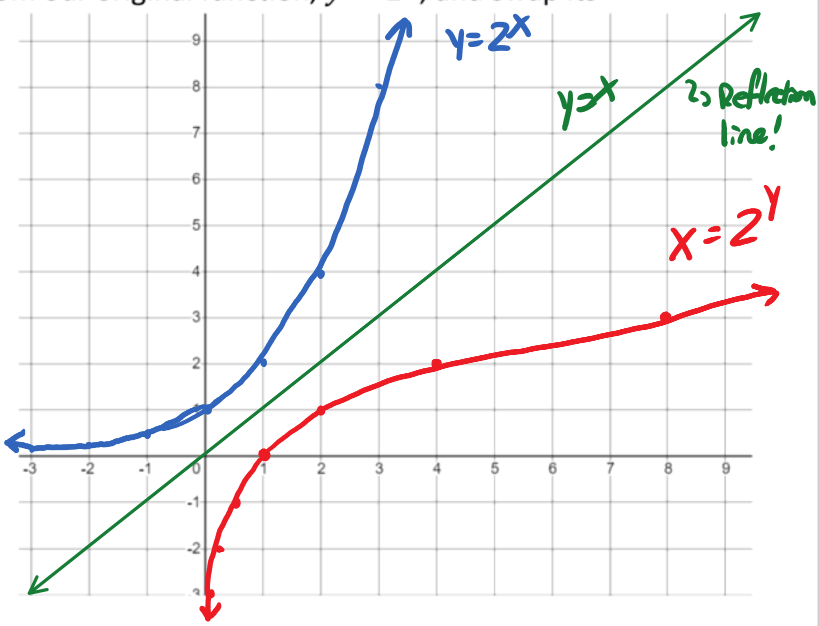
We can, however, get a bit of an idea of what the graph of the inverse will look like. All we need to do is take some points from our original function, $y = 2^x$, and swap its coordinates to graph the new function, $x = 2^y$.

Graph of $y = 2^x$

x	−3	−2	−1	0	1	2	3	4
y	1/8	1/4	1/2	1	2	4	8	16

Graph of $x = 2^y$

x	1/8	1/4	1/2	1	2	4	8	16
y	−3	−2	−1	0	1	2	3	4



Notice how both the original function and it's inverse both have 2 as the base.

The inverse of exponential functions are called " logarithmic function "

The inverse of the exponential function with base 2, such as $y = 2^x$, is a logarithmic function also with base 2, written as $y = \log_2 x$

That is to say, $x = 2^y \leftrightarrow y = \log_2 x$

When taking the inverse of an exponential function, we write it in logarithmic form.

Generally, logarithmic functions are in the form $y = a \log_b x$

where y = the value of the logarithm
a = a coefficient
b = the base of the logarithm (must be larger than 0, and not equal to 1)
x = the argument of the logarithm (must be larger than 0)

Although we won't spend time solving logarithms, it's good to understand how to compute basic logarithmic expressions.

To do so, we must remember the inverse relationship given above, written more generally: if $\log_b x = y$, then $b^y = x$

Example

Compute the value of the following logarithms:

$\log_2 8 =$ → $2^? = 8$
$2^3 = 8$, so
$\log_2 8 = 3$

$\log_4 16 =$ → $4^? = 16$
$4^2 = 16$, so
$\log_4 16 = 2$

If you use a graphing calculator (such as desmos.com) to graph logarithmic functions, some may not allow you to specify the base of the logarithm. To get around this, you can graph $y = \frac{\log(x)}{\log b}$ where b is the base.

Desmos: Go to "f(x)"
↳ $\boxed{\log_a x}$

Exploring the value of *b* in logarithmic functions

Given the general form of a logarithmic function, $y = \log_b x$, let's see what happens when we increase or decrease the value of *b*. We'll start by graphing the function $y = \log_3 x$. Notice how it passes through the point (1,0), as well as (3,1)

i) $y = \log_5 x$ passes through

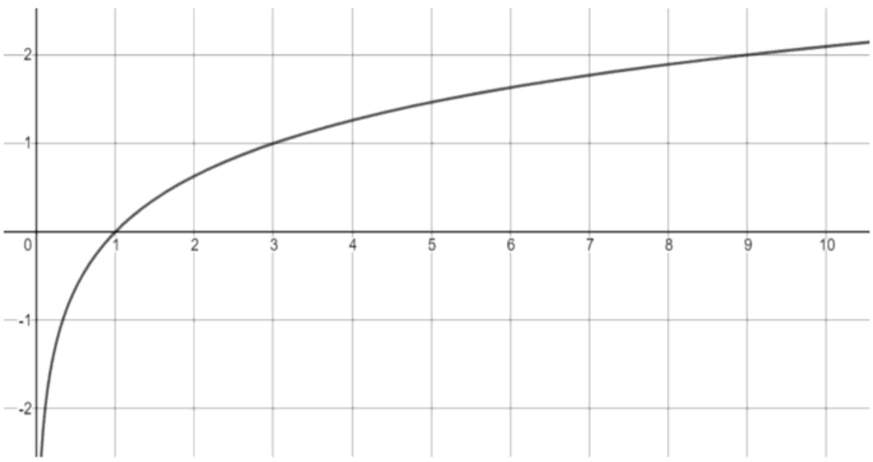
(1,) and (5,).

ii) $y = \log_{\frac{1}{3}} x$ passes through

(1,) and $\left(\frac{1}{3},\ \right)$.

iii) $y = \log_{\frac{1}{5}} x$ passes through

(1,) and $\left(\frac{1}{5},\ \right)$.



As we can see, logarithmic functions will always pass through the point (1,0) (because any base (b) raised to the power of 0 (y) will always equal 1).

Example

Determine the following characteristics of the function $y = \log_4 x$

Domain:

Range:

x-int. :

y-int. :

Asymptote:

x-value when y = 1 :

There exists two special logarithms.

The first one is known as a "Common logarithm". It has a base of 10. They are used so often that we don't even write the ten in to the expression.

ie. $y = \log_{10} x$ is simply written as $y = \log x$. That is to say, if a base is not specified, we assume it is base 10.

The second one is known as a "natural logarithm". It has a base of *e*. Remember from the last section that *e* is an irrational number approximately equal to 2.718 .

If a logarithm has base *e*, we write it as follows: $y = \log_e x \leftrightarrow y = \ln x$, this is read as "y is equal to *lawn* of x."

Exploring the value of *a* in logarithmic functions $y = a \log_b x$

Similar to changing the value of the logarithm's *b*-value, changing the *a*-value will alter the graph's shape.

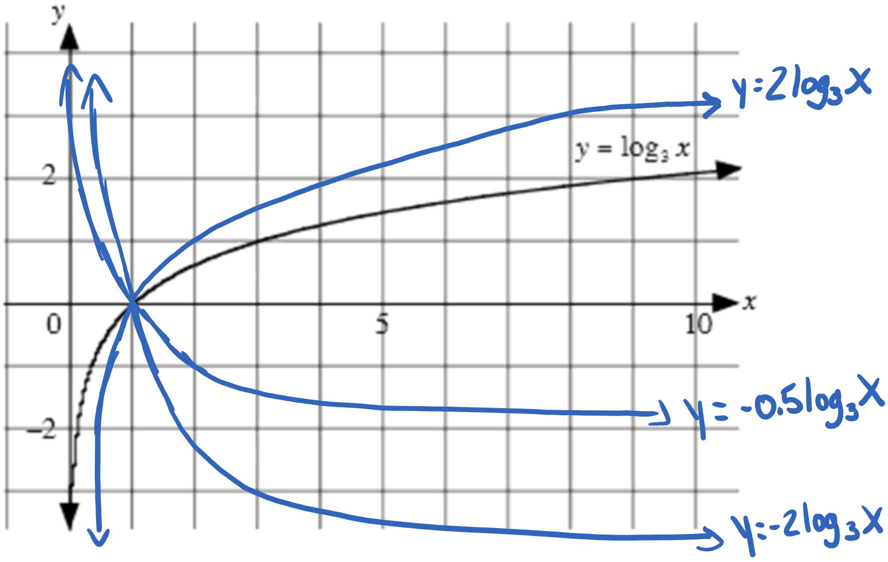
On the next page I've graphed the function $y = \log_3 x$ (a = 1), and we'll change the *a*-value while keeping *b* = 3.

We'll use a graphing calculator to help us graph the other functions listed below.

i) $y = \log_3 x$ **ii)** $y = 2\log_3 x$ **iii)** $y = -2\log_3 x$ **iv)** $y = -0.5\log_3 x$

b) Does changing the value of a in the graph of $y = a\log_b x$ alter

- the domain? no
- the range? no
- the x-intercept? no
- the y-intercept? no
- the asymptote? no



Notice how having a negative *a*-value flips the graph. So did having a *b*-value that was less than 1. If there are both a negative *a*-value and a *b*-value that is less than 1, the effect is that they "cancel" each other out, and the resulting graph is unchanged.

$y = a\log_b x$	$a > 0$	$a < 0$
$b > 1$	The graph increases from quadrant 4 to quadrant 1.	↓ From Q1 → Q4
$0 < b < 1$	↓ from Q1 → Q4	↑ from Q4 → Q1

Characteristics of the Graph of the Logarithmic Function $y = a \log_b x$

The following summarizes the basic characteristics of the graph of the logarithmic function with equation $y = \log_b x$, where $b > 0$ and $b \neq 1$.

Use the information from the previous explorations to complete the following.

- The x-intercept is 1.
- There is no y-intercept.
- The domain is $\{x \mid x > 0, x \in \mathbb{R}\}$.
- The range is $y \in \mathbb{R}$.
- The y-axis is a vertical asymptote with equation $x = 0$.
- The values of a and b determine whether the graph is increasing or decreasing.
  - When $a > 0$ and $b > 1$, the graph ↑ from quadrant 4 to quadrant 1.
  - When $a > 0$ and $0 < b < 1$, the graph ↓ from quadrant 1 to quadrant 4.
  - When $a < 0$ and $b > 1$, the graph ↓ from quadrant 1 to quadrant 4.
  - When $a < 0$ and $0 < b < 1$, the graph ↑ from quadrant 4 to quadrant 1.
- $y = \log_b x$ is equivalent to $x = b^y$, and is the inverse of $y = b^x$.
- b is the base of both the logarithmic function and the exponential function.