4.4 - Analyzing Graphs of Exponential & Logarithmic Functions

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There are many applications of exponential and logarithmic functions in real life.

Have you ever heard someone say something "increased exponentially"? As you might guess, this relates to how exponential functions work. Typical applications include modelling the growth of populations of people as well as bacteria.

Logarithmic functions have many applications as well. In science class, you may have heard that something is calculated on a "logarithmic scale" instead of a linear one. Some examples that use a logarithmic scale are the Richter scale which measures the magnitude of earthquakes, the pH scale which measures how acidic or basic a substance is in chemistry, and the decibel scale which measures how loud something is.

As we move forward, let's recall the general form of exponential and logarithmic functions:

Exponential Functions: $y = ab^x$ $, where\ a \neq 0, b > 0, x \in \mathbb{R}$

Logarithmic Functions: $y = a\log_b x$ $, where\ x > 0, x \in \mathbb{R}, b > 0, and\ b \neq 1$

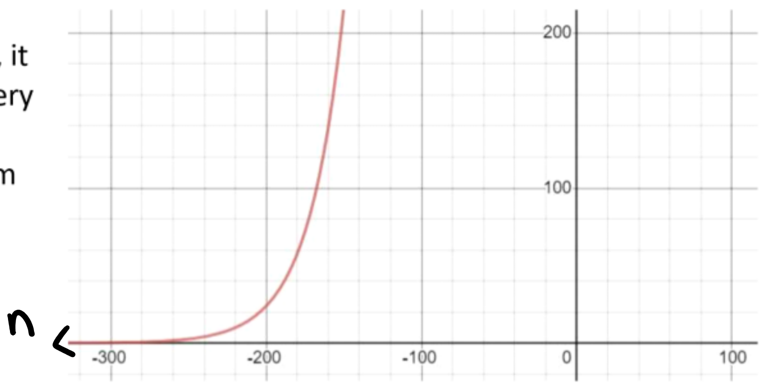
In 2012, the university population of a country was 160 000 and was increasing at an annual rate of 4.5%. The growth in the university population can be represented by the exponential function $P = 160\,000(1.045)^n$, where P represents the university population of the country as a function of the number of years, n, since 2012.

a) If the function representing the population is a "disguised" version of the form $y = ab^x$, state the values for a and b.

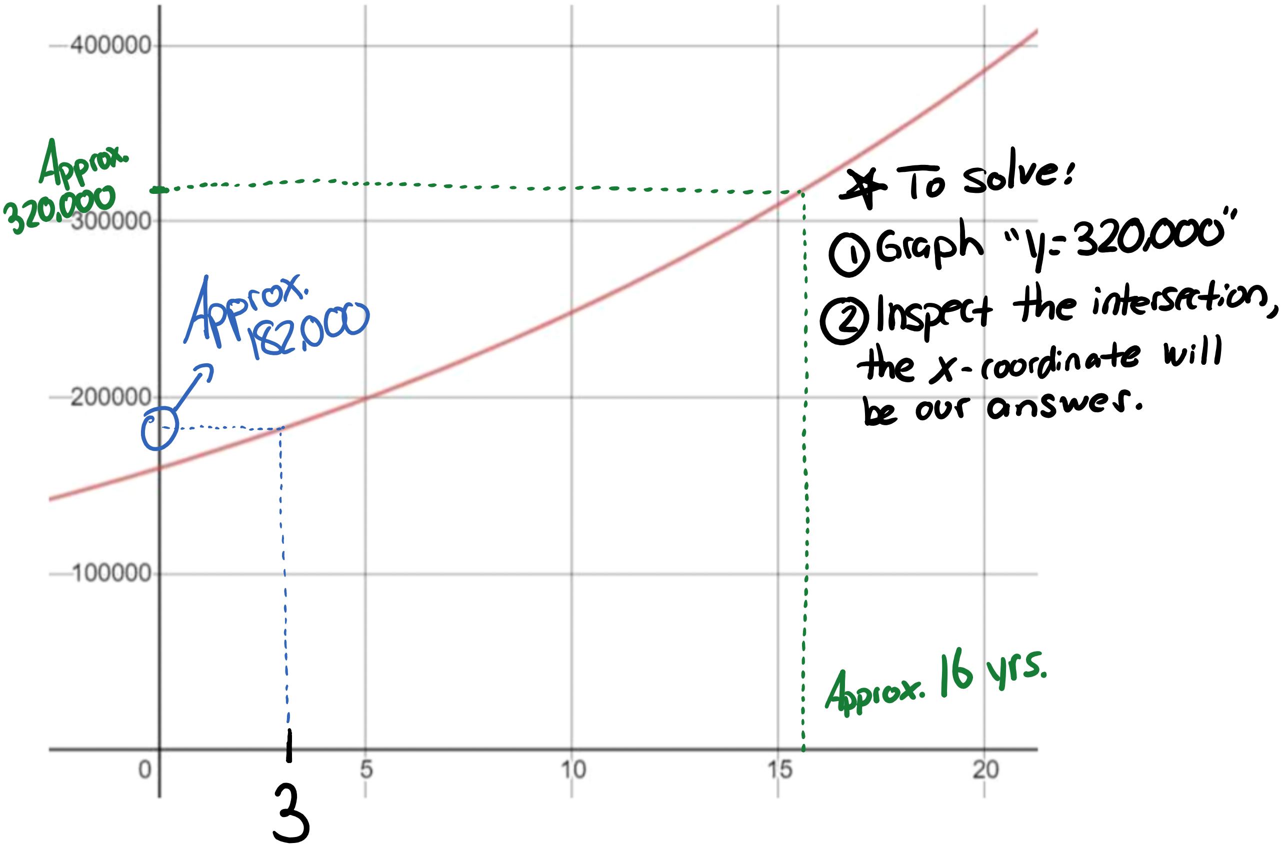
a=160,000 b= 1.045

b) Use a graphing calculator to graph the function which will show the population growth from 2012 to 2040. ~~State an appropriate window.~~

Graphing the function initially, it looks like the following. Not very useful since the years are negative, so we'll have to zoom in to where it's positive.



If we zoom in, we'll get something like this:



$P = 160000(1.045)^n$

$P = 160000(1.045)^3$

$P = 182{,}587$ people

c) Use the features of a graphing calculator to determine

i) the population in the year 2015

"n" is years since 2012, so in 2015, n = 3.

ii) the number of years required, to the nearest year, for the population to double from its 2012 size, assuming the population continues to grow at an annual rate of 4.5%.

160,000 people in 2012

Double → 2(160,000)

= 320,000

$P = 160{,}000(1.045)^n$

$\frac{(320{,}000)}{160000} = \frac{160000(1.045)^n}{160000}$

$2 = 1.045^n$

$\Rightarrow n = \log_{1.045} 2 = 15.74\ldots$ yr

So 16 yr.

The number of fish in a lake is decreasing by 5% each year as a result of overfishing. The number of fish present after t years is given by the formula $N(t) = N_0(0.95)^t$, where N_0 represents the initial population and $N(t)$ represents the final population.

2500

a) What type of function is represented by the above formula?

Exponential Decay

b) There were 2500 fish present in June 2012.

i) Write the equation of the function which could be entered into a graphing calculator to represent this scenario.

$N_0 = 2500$ (initial population) $y = 2500(0.95)^x$

ii) Explain how to use a calculator graph to determine how many fish you would expect to be present in June 2017. Determine the expected number.

June 2012 → June 2017: $t = 5$

- Graph $y = 2500(0.95)^x$
- Graph $x = 5$
- Where they intersect gives us .... 1934 fish

c) How many years, to the nearest tenth, would it take for the fish population to reduce to half of the number in June 2012?

Started with 2500
$2500 \div 2 = 1250$

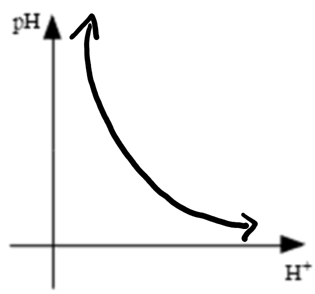
$N(t) = 2500(0.95)^t$
$(1250) = 2500(0.95)^t$
$0.5 = 0.95^t$
$t = \log_{0.95}(0.5) \doteq 13.5$ yr.

or: see where the graph intersects with $y = 1250$... $t \doteq 13.5$ yr.

In chemistry, the pH of a solution is defined as $pH = -\log[H^+]$, where the $[H^+]$ is the hydrogen ion concentration (expressed in moles/litre).

a) On the grid, sketch a graph of pH (y-axis) and hydrogen ion concentration (x-axis) using the window ~~x:[10^{-6}, 10^{-3}, 1] y:[2, 7, 1]~~

ie. $y = -\log x$



b) Determine the pH of a solution, to the nearest tenth, if the hydrogen ion concentration is 3.4×10^{-4} mol/L.

$[H^+] = 3.4 \times 10^{-4} \rightsquigarrow pH = -\log(3.4 \times 10^{-4}) = 3.5$

c) A patient gave a urine sample which was found to have a pH of 5.7. What was the hydrogen ion concentration? Answer in scientific notation using one decimal place.

- Graph $y = -\log[H^+]$
- Graph $y = 5.7$
- Intersect at... (5.7, 0.000001995)

∴ H^+ concentration is 1.995×10^{-6} mol/L $= 2.0 \times 10^{-6}$ mol/L

Carbon Dating

The decay of radioactive elements can sometimes be used to date events from the earth's past. In a living organism, the ratio of radioactive carbon, carbon-14, to ordinary carbon remains fairly constant. However, when the organism dies, no new carbon is ingested and the proportion of carbon-14 decreases as it decays.

An estimate of the age, x years, of organic remains can be determined from the exponential function $P = 100\left(\frac{1}{2}\right)^{\frac{x}{5730}}$, where P is the percentage of carbon-14 remaining.

a) State a suitable domain and range for the exponential function.

b) Graph the function using a calculator window x:[0, 20 000, 10 000] y:[0, 100, 50].

c) On May 1, 2012, a family in Whitehorse was digging up their basement when they discovered what were believed to be the remains of a prehistoric bison. An online article the same day in the Globe and Mail (see website below) indicated that paleontologists thought that the bones might be about 10 000 years old but that carbon dating results were required for verification. (http://www.theglobeandmail.com/news/national/shareTweet/article2419618/)

If the skeleton of the bison is about 10 000 years old, what percentage, to the nearest whole number, of carbon-14 should the paleontologists find remaining in the bones that were found?

d) A bone fragment with the carved image of a mammoth was discovered in the southern U.S. state of Florida in June 2011.

If the carbon dating test indicated that approximately 20.3% of carbon-14 was left, estimate the age of the bone fragment to the nearest 1 000 years.