4.5 - Modelling Data Using Exponential & Logarithmic Functions

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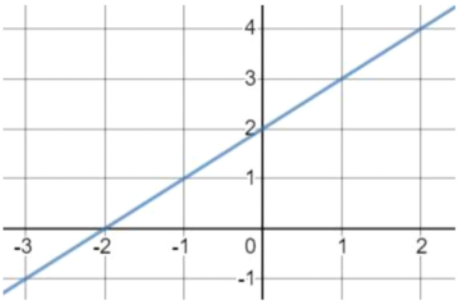
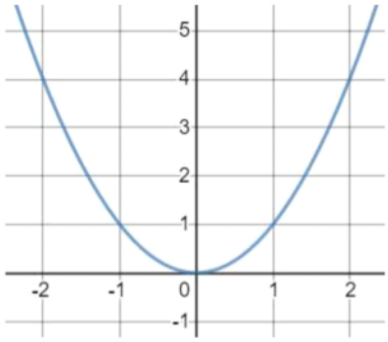
We've been graphing relations in math for several years. As long as there are two things that are related, we can plot them.

As you can imagine, when someone is studying a relation such as a scientist doing research, or a technician examining a sample, what they end up plotting can look very close to an equation we've studied:

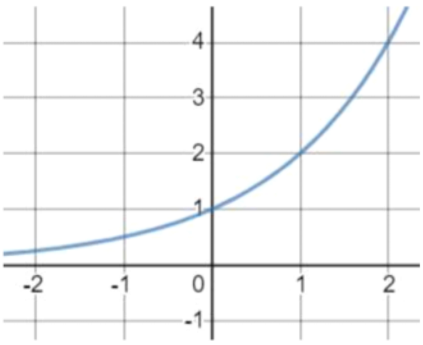
Linear



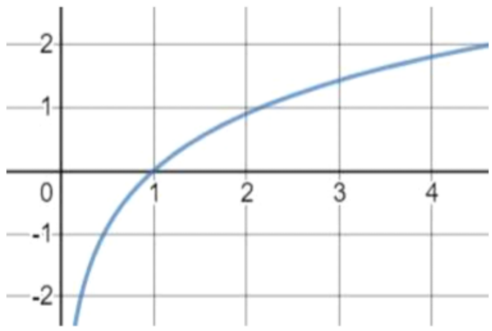
Quadratic



Exponential

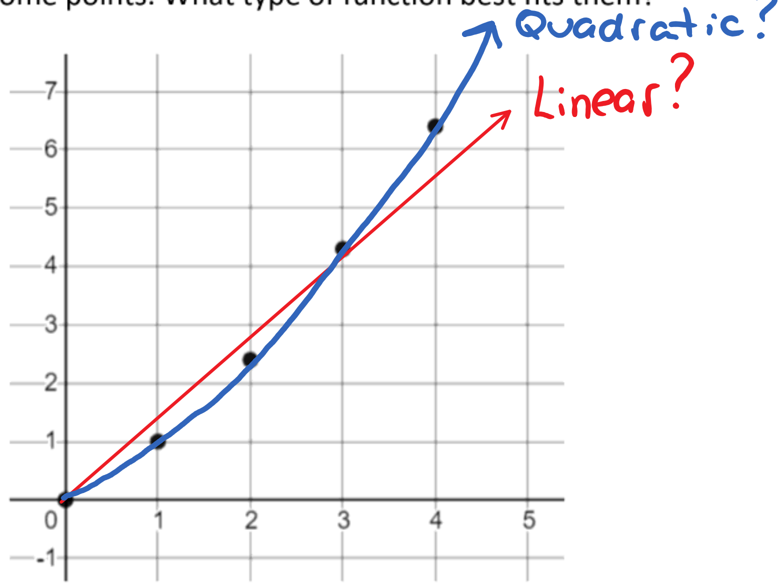


Logarithmic



Now that we have a few functions in our toolkit, the question becomes which one best fits in to the plots we might encounter?

Let's take a look at some points. What type of function best fits them?



There's a branch of mathematics that deals strictly with fitting functions to data that appears in real life. As we've seen, changing values of "a" and "b" changes the shape of the functions we've studied. By changing these values, we can try to make these functions better fit the points we're dealing with.

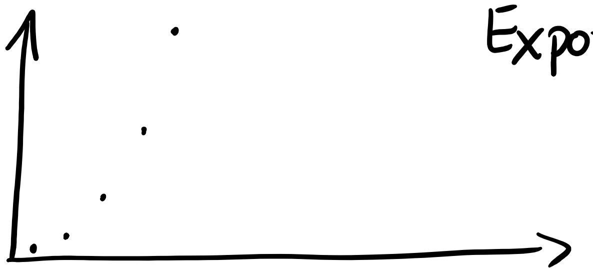
This process is called regression. Doing regression by hand is beyond what we're going to learn in this course, but there's lots of technology that can do this for us. Graphing calculators can do it, Excel can do it, and Desmos can do it.

Example

Consider the following data points:

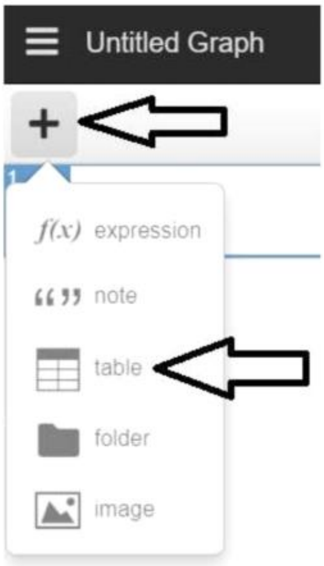
x (Time)	1	3	5	7	9
y (Speed)	2.03	6.12	14.25	30.90	68.11

Make a quick sketch of the graph. What function appears to represent the shape of the data?

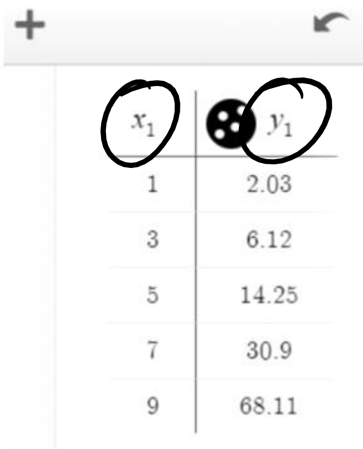


Exponential?

Let's go in to Desmos and enter our data. We'll need to create a table:



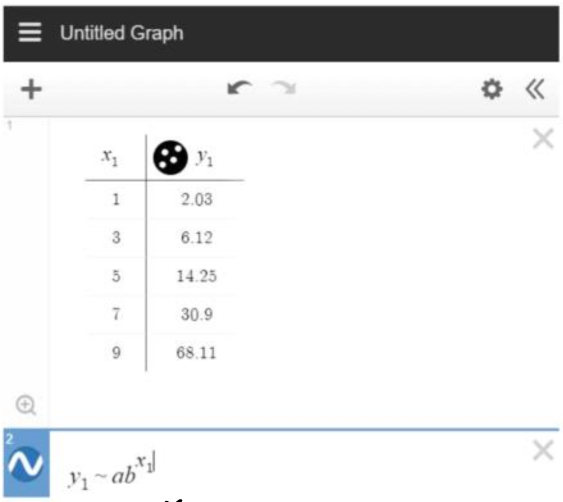
From there, we can enter our data points:



Notice how Desmos calls the variables x_1 and y_1. We'll use these two variables to create our function (regression).

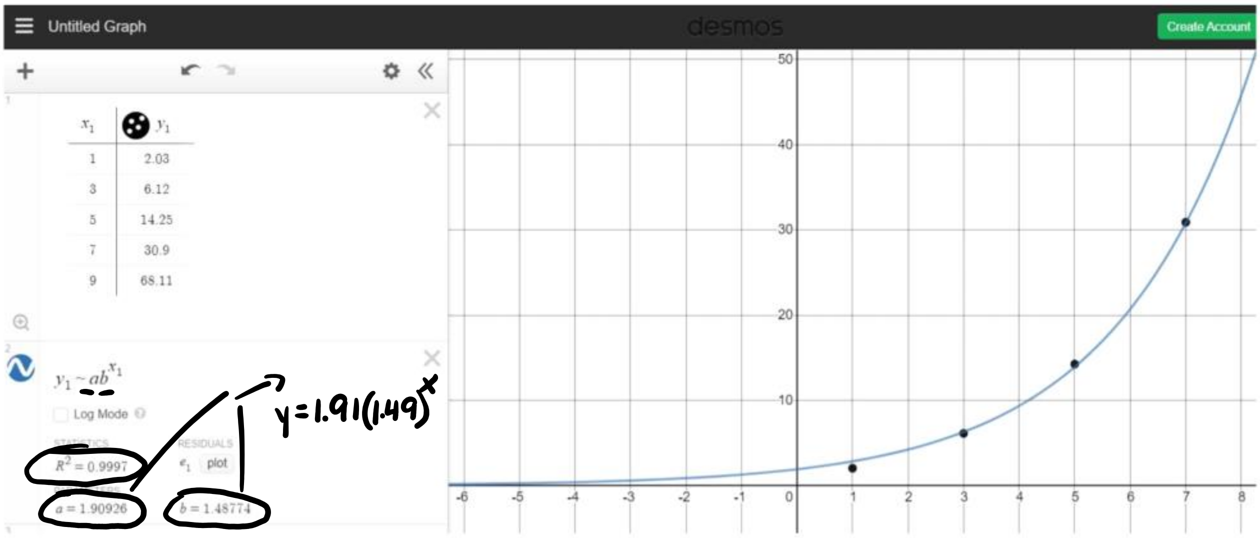
Since we decided this looks like an exponential function, we'll enter the general form of an exponential equation, but instead of x and y, we'll enter x_1 and y_1, and instead of using "equals", we'll use a tilde ("~"):

$\cong$



$y = ab^x$

Once we do that, Desmos will populate the following information:



As you can see under the "Parameters" section, Desmos has calculated the "a" value to be equal to 1.90926, and the "b" value to be 1.48774.

This means the exponential function which best fits the data points is:

$$y = 1.90926(1.48774)^x$$

As you can see, Desmos also plots the line. As you can also see, the line is not perfect, that is to say the points don't pass *exactly* through the line, but they're close.

In fact, the "R^2" value listed there tells us how close we are. An R^2 value of 1.0000 means the line perfectly fits the data we were given.

In Class Ex. #4 from the previous lesson we discussed carbon dating. The percentage of carbon-14 remaining was expressed as an exponential function of the age of the organic remains.

Since a logarithmic function is the inverse of an exponential function, it is possible to switch the variables and write the age as a logarithmic function of the percentage of carbon-14.

The table below shows data for five recently discovered fossils.

% carbon-14 (x)	95	79	68	38	27
Age in years (y)	425	1 950	3 191	8 000	10 824

a) Create a scatterplot using a graphing caclulator to confirm that a logarithmic function is an appropriate model for the data.

~~**b)** Use the natural logarithm regression feature of a calculator (LnReg) to determine a function in the form $y = a + b \ln x$ which models the data. Use integer values for a and b.~~

c) On a graphing calculator, display the graph of the regression equation in b) over the scatterplot in a).

$y = ab^x \rightsquigarrow y = 26840(0.968)^x$

d) Use the graph in c) to answer the following questions:

i) A bone fragment with the carved image of a mammoth was discovered in the southern U.S. state of Florida in June 2011. If the carbon dating test indicated that approximately 20.3% of carbon-14 was left, estimate the age of the bone fragment to the nearest 1 000 years.

$\Rightarrow x = 20.3$

$y = 26840(0.968)^{(20.3)}$

$= 13{,}869.22\ldots \cong 14{,}000$ years

ii) If the skeleton of a bison is about 10 000 years old, what percentage, to the nearest whole number, of carbon-14 should the paleontologists find remaining in the bones that were found?

$\Rightarrow y = 10{,}000$

$\frac{(10{,}000)}{26840} = \frac{26840(0.968)^x}{26840}$

$0.37258 = 0.968^x \rightsquigarrow x = \log_{0.968} 0.37258$

$x \doteq 30\ \%$

iii) Compare the answers with Class Ex. #4 in the previous lesson.