2.1 - The Fundamental Counting Principle

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Unit 1: Permutations & Combinations

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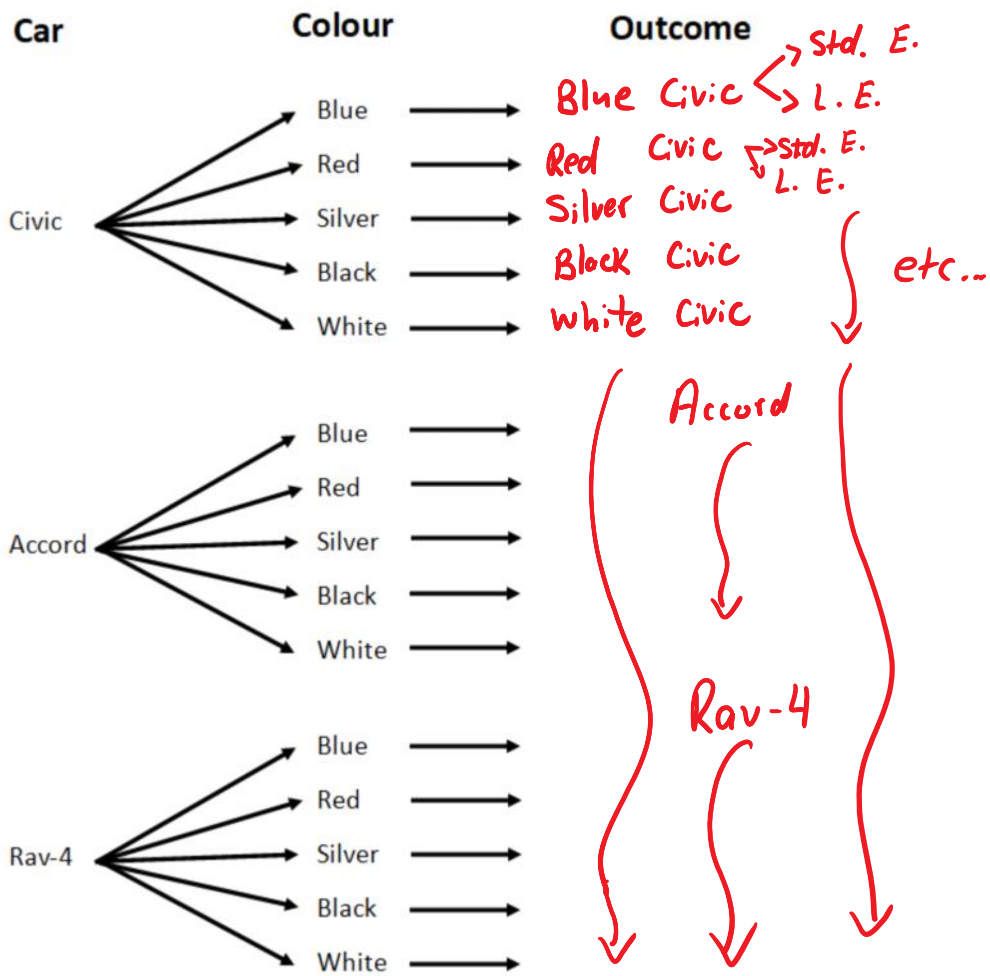
We're going to begin this chapter with an exercise to illustrate the "fundamental counting principle".

Chester's shopping for a new car. He's narrowed down his choices to 3 models: The civic, the accord, and the rav-4. The salesperson tells him that each vehicle is available in 5 different colours: blue, red, silver, black, and white.

If we consider each colour to be a different "choice," how many choices does Chester have?

To help us out, we'll use a tree diagram to organize the possibilities:



So we can say Chester has ___15___ choices.

The salesperson tells Chester each model in any colour is available in the standard edition, or the leather edition. How many choices does Chester have now? Extend the diagram to show your work.

Chester now has ___$15 \times 2 = 30$___ choices.

The Fundamental Counting Principle

The fundamental counting principle states that if the number of choices for the first stage of some task is "*a*," the number of choices for the second stage of the same task is "*b*," and the number of choices for the third stage of the same task is "*c*," etc., then the total number of ways that task can be completed is:

$$\underline{Total\ Possibilities = a \times b \times c \times \ldots}$$

In the previous example, the first stage was selecting the model we wanted. That stage could be completed in ___$a = 3$___ different ways, one for each model. The second stage was the colour which could be completed ___$b = 5$___ ways, one for each colour. Finally, the edition to be selected was the third stage, and it could be completed ___$c = 2$___ ways, one for each edition.

We can say that:

a = ___3___

b = ___5___

c = ___2___

So the total number of possibilities are:

$a \times b \times c$

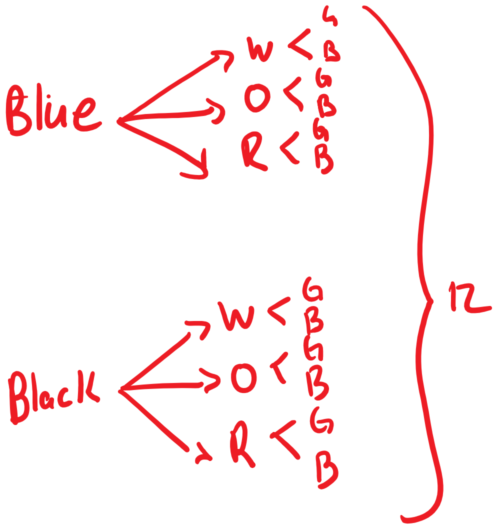
$3 \times 5 \times 2 = 30$ possibilities.

Which is the answer we arrived at using our tree diagram.

Example

A toy manufacturer makes a toy in three parts:
Part 1: The top part may be blue or black
Part 2: The middle part may be white, orange, or red
Part 3: the bottom part may be green or blue

Determine the total number of possible colour combinations for the toy by first using a tree diagram, then use the fundamental counting principle.



3 total Stages		# of choices
a = top	:	2
b = mid	:	3
c = bottom	:	2

$\Rightarrow$ total possibilities $= a \times b \times c$
$= 2 \times 3 \times 2$
$= 12$

Sometimes you will be restricted in your options for the number of possibilities in a given stage of a task. It's a good idea to deal with those stages first.

For example, let's say I ask you to determine how many odd 4-digit numbers there are. You can set up each digit of the number as a stage, while keeping in mind that a number such as "0443" is not acceptable, as it would be a 3-digit number.

There are ten available possibilities for each digit (0-9), but in certain stages, we are restricted to less.

I'll fill in one of the restricted stages to begin:

Note: not the number "5", this is 5 possibilities for this digit.

____ ____ ____ __5__

Why are there only five possible numbers for the last digit?

For the entire number to be odd, it must end in an odd number $\underbrace{(1, 3, 5, 7, 9)}_{\text{5 possibilities}}$

There's one more stage that's restricted in this question. I gave you a hint at the start of the question:

__9__ ____ ____ __5__

possibilities

Now we can fill in the rest. Use the fundamental counting principle to determine the total number of odd 4-digit numbers.

__9__ __10__ __10__ __5__

$$\# \text{ of possibilities} = a \times b \times c \times d$$
$$= 9 \times 10 \times 10 \times 5$$
$$= 4500 \text{ odd 4-digit numbers.}$$

Now what if I ask you the same question, but this time I want the number to have no repeating digits (no number can be used more than once).

We can reuse the same first step of the above solution, but not the second. Why?

Still an odd number

Can't use 9 again for the first digit because we cannot repeat the digit we used for the last digit.

∴ We have one less possibility (8)

Let's use the fundamental counting principle to complete the solution:

8 8 7 5 → 8 × 8 × 7 × 5

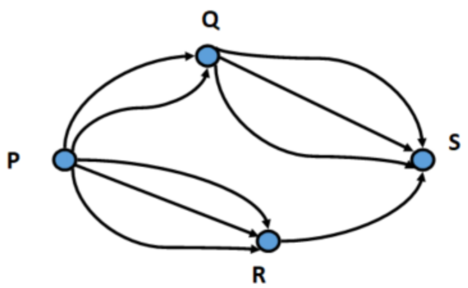
cant be 0 down 2 choices down 3 choices odd #

= 2,240 possibilities.

Sometimes, you'll have to use two separate instances of the fundamental counting principle and add them together at the end. Typically, you'll have to do this when you see the keyword "**or**" in the question.

Example

There are two routes from Pitland (P) to Queensville (Q), three routes from Queensville to St. Lukes (S), three routes from Pitland to Rutherford (R), and one route from Rutherford to St. Lukes as shown by the following diagram:



How many routes are there from P to S going through Q?

P→Q: 2 routes = a

Q→S: 3 routes = b

P→S through Q

a × b = 2 × 3

= 6 possible routes

How many routes are there from P to S going through R?

P→R: 3 routes = a

R→S: 1 route = b

a × b = 3 × 1 = 3 possible routes.

How many routes are there going from P to S?

6 + 3 = 9 possible unrestricted routes.