2.5 - Combinations Pt. I

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FOUNDATIONS OF MATH 12 Ch. 2 – Day 5: Combinations Pt. I

Let's say students in English 12 were required to read three different books for the course: Book A, book B, and book C.

Jo had a few late returns, so he can only take out one book at a time, but Sophie can take out as many as she wants.

List the possible order of books Jo and Sophie can take out.

Jo	Sophie
A, B, C A, C, B B, A, C B, C, A C, A, B C, B, A (3!)	A, B, C all at once.

Although the number of different arrangements are the same, after the books are signed out for Sophie in each arrangement, is she left with a different result?

No, she has all of her books.

This is an example where the order of selection doesn't matter. When the order doesn't matter, we call it a combination.

Where the order of selection does matter, it's a permutation (which we've already learned about).

From the example above, although there were six possible *permutations*, because the order didn't matter with what Sophie ended up with, there was one *combination*.

Essentially, we take the answer from the permutation formula, but we have to divide by the factorial of how many options we're taking so we don't double count them.

The number of Combinations of "n" Different Elements Taken "r" at a Time is:

"n choose r"

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

Example

Five different students are running the 100m event. In what order is it possible for the gold, silver, and bronze medals to be awarded? Is this a permutation or a combination?

Order matters because you get a different medal for 1st, 2nd, 3rd.

∴ Permutation.

5 students running
$n=5$
Order of top 3
$r=3$

$\Rightarrow {}_5P_3 = \frac{n!}{(n-r)!} = \frac{(5)!}{(5-3)!} = \frac{5!}{2!}$

$= 60$ possibilities

How many ways can the students finish in the top three? Is this a permutation or a combination?

Order that you finish in for top 3 DOESN'T MATTER.

∴ combination.

$n=5$
$r=3$

$\Rightarrow {}_5C_3 = \frac{n!}{(n-r)!r!} = \frac{(5)!}{(5-3)!(3)!}$

$= \frac{5!}{2!3!} = 10$

∴ 10 possible top 3 combinations

Example

Four students from a class of 11 are to be chosen to go on a field trip. In how many ways can they be selected? Is this a combination or a permutation?

The order the students are picked doesn't matter because so long as they're selected, their outcome doesn't matter.

∴ Combination.

$n=11$
$r=4$

nCr

$\Rightarrow {}_{11}C_4 = \frac{11!}{(11-4)!4!}$

$= 330$ possible combinations

Example

Poker is a game of cards played with a standard 52-card deck with hands of 5 cards dealt to each player. How many possible poker hands are there? Is this is a permutation or combination?

The order in which you recieve the cards does not change what "hand" you have.
∴ combination.

$n = 52 \Rightarrow {}_{52}C_5 = \frac{52!}{(52-5)!5!}$
$r = 5$

$= 2,598,960$ possible poker hands.

How many hands is it possible to have 3 kings and 2 queens?

order doesn't matter,
∴ combination.

4 possible Kings ⇝ ${}_4C_3$
4 possible queens ⇝ ${}_4C_2$

$\Rightarrow {}_4C_3 \times {}_4C_2$

$\frac{4!}{(4-3)!3!} \times \frac{4!}{(4-2)!2!}$

$= 4 \times 6$

$= 24$