# 6.1 - Polynomial Functions of Degrees 0, 1 and 2

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# Unit 5: Polynomial & Sinusoidal Functions

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We've seen polynomial functions in previous math courses.

As a review, polynomials consist of one or more terms which are separated by addition or subtraction.

ie. $3x^2 - 2x + 1$ $2n + 4$ $4$ are all polynomials

The degree of a polynomial is the value of its largest exponent, and if the polynomial has a term with no variable, it is called the constant term.

Example

Complete the following.

- The degree of $f(x) = x^2 - 4x - 5$ is 2. The constant term is -5.
- The degree of $f(x) = 2x - 4$ is 1. The constant term is -4.
- Since $f(x) = 3$ can be written as $f(x) = 3x^0$, the degree of $f(x) = 3$ is 0.

The number that multiplies a variable is called a coefficient, and the
* leading coefficient is the number that multiplies the term with the largest exponent.

Example

Complete the following.

- The leading coefficient of $f(x) = x^2 - 4x - 5$ is 1.
- The leading coefficient of $f(x) = 2x - 4$ is 2.

In a polynomial, all the exponents must be whole numbers.

Example

Write a polynomial function that satisfies the following conditions.

- degree 2, leading coefficient –3 $f(x) = -3x^2 + 7$.
- degree 2, leading coefficient 7, two terms $f(x) = 7x^2 - 3$.
- degree 1, leading coefficient 1 $f(x) = x$.

### ❖ Polynomial Functions of Degree 0

If a polynomial function has a degree 0, that means it's largest exponent on a variable is zero. Anything raised to the power of 0 is equal to 1, so let's see what happens to the following degree 0 polynomial when we simplify it:

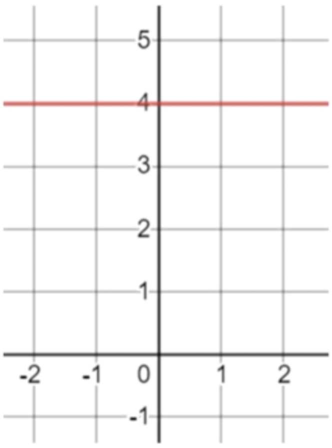
$$f(x) = 4x^0$$

$$= 4(1)$$

$$= 4$$

As you can see, the "x" cancels out and we're left simply with a constant. Therefore, **any polynomial function without a variable is a degree 0 polynomial function.**

The above function is graphed below:




As you can see, the line that's produced is horizontal.

State the domain and range of the polynomial function:

D: $x \in R$

R: $\{y \mid y = 4\}$

How many y-intercepts does this graph have?

1

How many x-intercepts?

0

Is there a case where there will be an x-intercept(s)?

Yes. $f(x) = 0$ has infinite x-intercepts.

Generalize the above work to complete the following statements about **all** polynomial functions of degree zero in the form $f(x) = c$.

**a)** The graph is a horizontal line with a slope of 0 .

**b)** The domain is $x \in R$ .

**c)** The range is $\{y \mid y = c\}$ .

**d)** There are no $x$-intercepts, except for the function $f(x) =$ 0 .

**e)** There is 1 $y$-intercept.

**a)** Does the value of a polynomial function of degree zero depend on the value of $x$?

no

**b)** Since the values of a polynomial function of degree zero remain constant for all values of $x$, this type of function is called a constant function.

## ❖ Polynomial Functions of Degree 1

Degree one polynomial functions are also straight lines, but they are oblique (diagonal).

In this course, we will only be examining degree one polynomials in **slope-intercept** form:

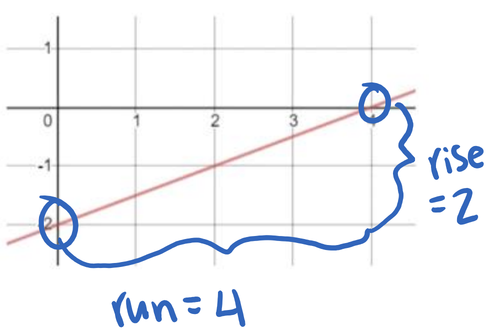
$$f(x) = mx + b$$

where m = slope, and b = the y-intercept.

Recall that the slope of a line is given by $m = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$

**Example**

Determine the slope of the following degree one polynomial function:




$m = \frac{rise}{run} = \frac{2}{4} = 1/2$

If the slope, m, of the function (which is also the leading coefficient) is positive, the graph will increase, and if it is negative, it will decrease.

Generalize the above work to complete the following statements about **all** polynomial functions of degree one in the form $f(x) = mx + b$.

**a)** The shape of the graph is a line with a slope of m.

**b)** The domain is $x \in R$. **c)** The range is $y \in R$.

**d)** There is 1 $x$-intercept and 1 $y$-intercept. **e)** The $y$-intercept is b.

**f)** The direction of the line is determined by the value of the leading coefficient.

II | I
III | IV

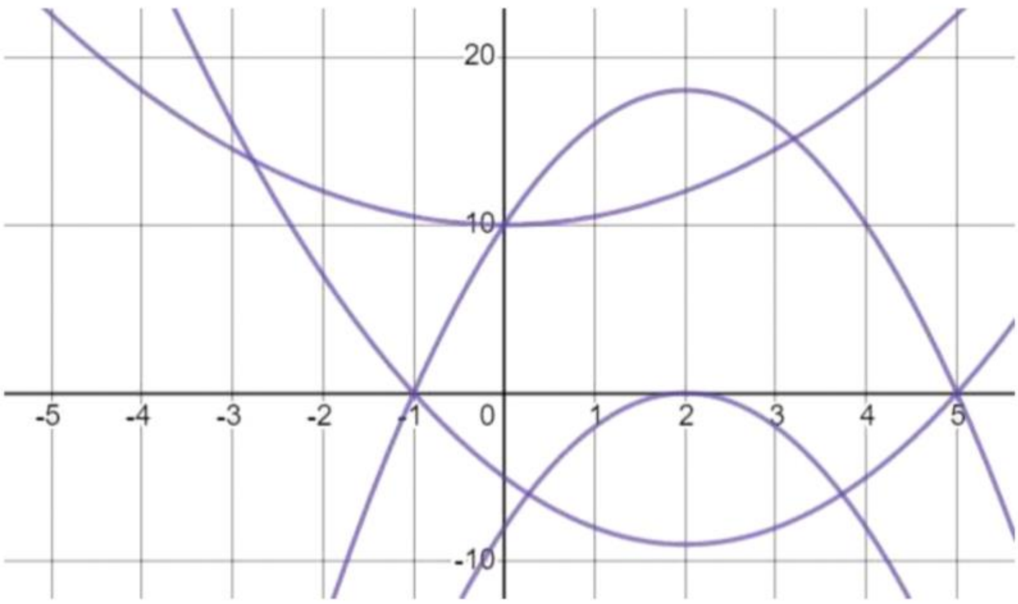
- **i)** If the leading coefficient is positive, the line slopes up from quadrant 3 to quadrant 1.
- **ii)** If the leading coefficient is negative, the line slopes down from quadrant 2 to quadrant 4.

**g)** Since the graph of a polynomial function of degree one is a straight line, this type of function is called a linear function.

❖ **Polynomial Functions of Degree 2**

Polynomial functions of degree two are called quadratic functions. We studied them last year.

Let's look at some example functions:

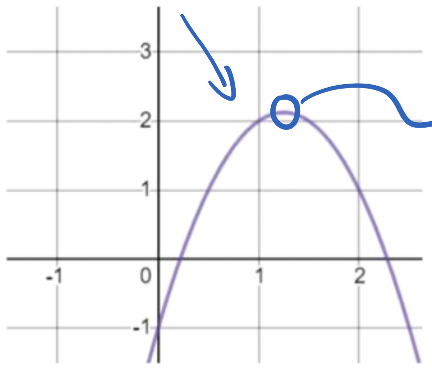


For the above functions, circle the properties that are the same for all graphs:

Domain, Range, Number of x-intercepts, Number of y-intercepts

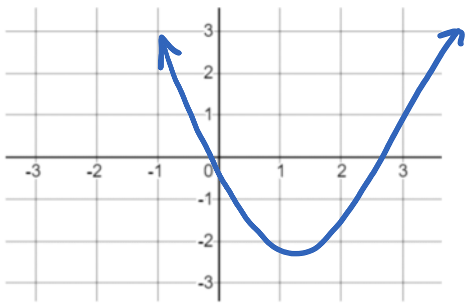
Recall that the shape these functions produced are called parabolas.

Each polynomial function of degree two has what we call a turning point which is also either a maximum, or a minimum:



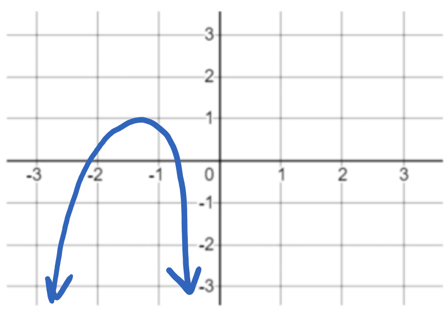
turning point

How does the leading coefficient help us predict the shape of the parabola?



opening up,

leading coefficient is positive.



opening down,

leading coefficient is negative.

$y = ax^2 + bx + c$

The graphs on the previous page show that polynomial functions of degree two can have 0, 1, or 2 $x$-intercepts. The $x$-intercepts are the same as the **zeros** of the function.

Three of the functions on the previous page can be written in factored form as shown below. Complete the following:

what makes them $=0$?

$x=-1$ $x=5$

**a)** The graph of $f(x) = x^2 - 4x + 5 = (x+1)(x-5)$ has 2 $x$-intercepts, at -1 and 5.

**b)** The graph of $g(x) = -2x^2 + 8x + 10 = -2(x+1)(x-5)$ has 2 $x$-intercepts, at -1 and 5.

$x=-1$ $x=5$

**c)** The graph of $h(x) = -2x^2 + 8x - 8 = -2(x-2)^2$ has 1 $x$-intercept at 2.

$x=2$

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Consider the functions $P(x) = 7(x-1)(x+6)$ and $Q(x) = -(x+3)^2$.

**a)** State the zeros of each function.

P(X) x-int's: 1, −6 Q(x): −3

**b)** Complete the table below with the characteristics of the graph of each function.

| | Direction of Opening | Number of $x$-intercepts | Value(s) of $x$-intercept(s) | Value of $y$-intercept |
|---|---|---|---|---|
| $P(x)$ | ↑ | 2 | 1, −6 | −42 |
| $Q(x)$ | ↓ | 1 | −3 | −9 |

when $x=0$

$7(0-1)(0+6) = -42$

$-(0+3)^2 = -9$

This concludes the review for polynomial functions of degrees 0 – 2 we've seen in previous courses. What we may have not discussed in previous classes is end behaviour of a function.

The end behaviour of a function describes the appearance of the function at the left and right tails of the line or curve. We might also call these tails the "extremes" of the function.

### ❖ End Behaviour of Degree Zero Polynomial Functions

The end behaviour of a degree zero function remains constant since it's just a horizontal line.

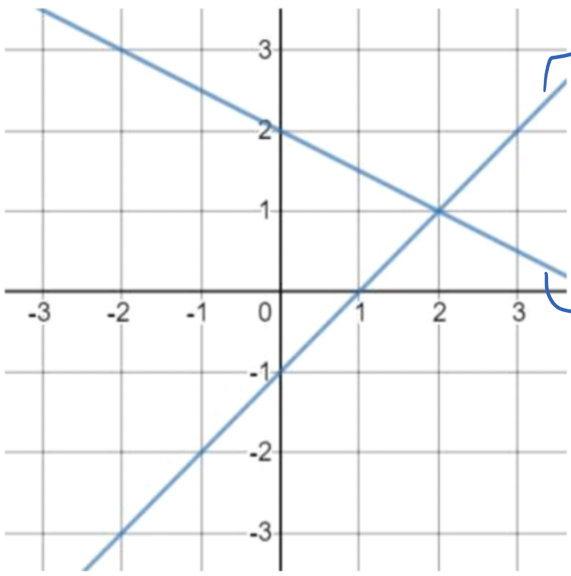
### ❖ End Behaviour of Degree One Polynomial Functions

For degree one polynomial functions, the end behaviour is determined by the leading coefficient of the function.

If the leading coefficient is positive, the left tail of the function goes down while the right tail of the function goes up. This is an increasing function.

If the leading coefficient is negative, the left tail of the function goes up while the right tail of the function goes down. This is a decreasing function.

ie.




(+)ve leading coefficient.

(−)ve leading coefficient

## ❖ End Behaviour of Degree Two Polynomial Functions

For degree two polynomial functions, the end behaviour is also determined by the leading coefficient.

If the leading coefficient is positive, both the left and right tails point up.

If the leading coefficient is negative, both the left and right tails point down.

### Example

Complete the table using the word "up" or "down" to describe the end behaviour of the graph of each function and the word "maximum", "minimum", or "none" to describe the nature of the turning point of each graph.

| | Left Tail | Right Tail | Nature of Turning Point |
|---|---|---|---|
| $f(x) = -2x + 4$ | ↑ | ↓ | none |
| $g(x) = 2x^2 - 7$ | ↑ | ↑ | minimum |
| $h(x) = -5 + 3x$ | ↓ | ↑ | none |
| $k(x) = 6 - 2x - x^2$ | ↓ | ↓ | maximum |