# 3.1 - Terminology & Notation

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# Unit 3: Probability

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In math, probability deals with chance or prediction. Before we can start, we need to understand some terminology:

- A trial is any operation whose outcome cannot be predicted with certainty.

  eg. Tossing a coin, or rolling a die.

- An experiment consists of one or more trials.

  eg. Spinning a spinner, rolling 2 dice, tossing a coin and rolling a die.

- An outcome is the result of an experiment.

  eg. "Heads", or "2 & 1"

- The Sample space of an experiment is some set (usually set $S$) which contains **all** possible outcomes.

  eg. For rolling a die, $S = \{1,2,3,4,5,6\}$

  For flipping a coin and spinning a spinner,

  $S = \{(H,1),(H,2), \dots ,(T,5),(T,6)\}$

- An event is a subset of the sample space. It represents one or more possible outcomes of an experiment.

  eg. {Heads}, or {3,6}

To denote the probably of something happening, let's say the probably of event "A" happening, we write *" P(A)"* which is read as "the probability of A."

Usually, probabilities are listed as decimals or fractions between 0 and 1, but percentages are sometimes used. ie. $0 \leq P(A) \leq 1$

If the event A does not include any element in the sample space, then A is impossible, and we write P(A) = 0. ie. P(rolling a 9) = 0.

If the event A includes all elements in the sample space, then A is certain, and we write P(A) = 1. ie. P(rolling a number 1 to 6) = 1.

If some experiment has a set of equally likely outcomes, then the probability of an event is given by:

$$P(A) = \frac{\#\ of\ outcomes\ that\ satisfies\ A's\ requirements}{total\ \#\ of\ possible\ outcomes}$$

**Example**

A fair six-sided die is rolled, what is the probability of rolling a 1?

i) What is the sample space?

$S = \{1, 2, 3, 4, 5, 6\}$

ii) What are the elements from the sample space which satisfy the event?

$\{1\}$

iii) Are all outcomes equally likely?

Yes, it is a "fair" die

iv) What is the probability of the event?

$P(A) = \frac{\text{outcomes that satisfy } A}{\text{total outcomes}} = \frac{1}{6}$, $P(A) = \frac{1}{6} \doteq 0.167$

**Example**

A fair spinner which contains "spades," "clubs," "diamonds," and "hearts" is spun. What is the probability it lands on "hearts"?

i) What is the sample space? $S = \{\text{spades, clubs, diamonds, hearts}\}$

ii) What are the elements from the sample space which satisfy the event?

$\{\text{hearts}\}$

iii) Are all outcomes equally likely?

Yes, it is a fair spinner.

iv) What is the probability of the event?

$P(A) = \frac{\#\text{ outcomes satisfying } A}{\text{total } \#\text{ outcomes}} = \frac{1}{4}$

$\therefore P(A) = \frac{1}{4} = 0.25$

Remember the complement from previous sections? The complement of something is whatever is left over from picking something else.

When discussing probability, it's no different. Consider the spinner from the previous example. If we consider event "A" to be "landing on hearts," the complement of A would be "does not land on hearts." They are called "complementary events."

The probability of two complementary events will always add to 1.

As always, we list the complement as $A'$ or $\overline{A}$ (read as "A bar") $\bar{A}$ $A^c$

ie. $P(A) + P(A') = 1$

**Example**

List the outcomes for the complement of "rolling a 1 on a six-sided die," and determine the probability for that complement.

A : rolling a 1 $= \{1\}$

A': not rolling a 1 $= \{2,3,4,5,6\}$

$\Rightarrow P(A') = \dfrac{\text{\# outcomes favourable to } A'}{\text{Total \# outcomes}}$

$= \dfrac{5}{6}$

We will discuss what are called **compound events**. Compound events are when more than one trial or more than one event are combined. For example, flipping two coins would be considered a compound event.

Let's use this example to explore compound events.

Billy decides to flip to two coins and study the resulting probabilities.

He proposes the following sample space:

$S = \{two\ heads, one\ head\ and\ one\ tail, two\ tails\}$ and he considers the favourable outcome to be "2 heads."

Using the formula

$$P(A) = \frac{\#\ of\ outcomes\ that\ satisfies\ A's\ requirements}{total\ \#\ of\ possible\ outcomes}$$

Billy then concludes $P(two\ heads) = \frac{1}{3}$

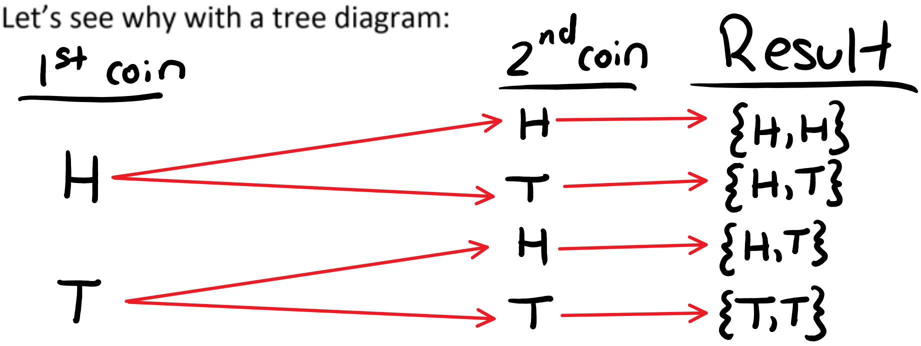
Billy is wrong. 😢

Why is Billy wrong?

He's assuming all outcomes are equally likely.

They are not.

Let's see why with a tree diagram:




Determine the following correct probabilities.

P(2 H) P(1H, 1T) P(2 T)

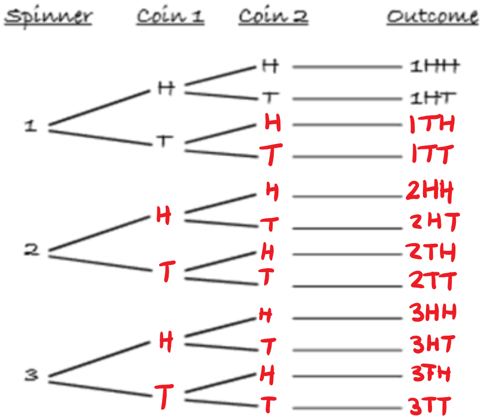
$P(2H) = \frac{1}{4}$ $P(1H, 1T) = \frac{2}{4} = \frac{1}{2}$ $P(2T) = \frac{1}{4}$

## Example




Consider an experiment of spinning an equally spaced triangular spinner numbered 1, 2, 3, and tossing two coins.

**a)** Complete the tree diagram to show all the outcomes for the experiment.




**b)** How many elements are in the sample space?

$n(S) = 12$

**c)** Show how you can use the fundamental counting principle to determine the answer to b).

$3 \times 2 \times 2 = 12$

**d)** Are all the outcomes equally likely?

Yes!

**e)** State the probability of obtaining :

**i)** a three and two heads

↳ 3HH

→ $\frac{1}{12}$

**ii)** a prime number and exactly one tail

↳ 2HT, 2TH, 3HT, 3TH

↳ $\frac{4}{12} = \frac{1}{3}$

## Example


A blue die and a red die are rolled. The outcome "3 on the blue die and 4 on the red die" can be represented by the ordered pair (3, 4).

**a)** Show all the possible outcomes in the array.

**b)** How many points are in the sample space?

**c)** List the event "the same number appears on each die" as a subset of the sample space.

Red Die

| Blue Die | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | (3, 4) | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

**d)** State the probability of the following events:

**i)** the same number appears on both dice **ii)** a different number appears on each die

**e)** How can the answer for d) ii) be determined using the answer in d) i)?