3.2 - Odds & Probability

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Theory isn't always reality.

When dealing with probability, often it's impossible to predict something with 100% certainty.

We'll now discuss the difference between theoretical probability and experimental probability.

Theoretical probability is given by the formula:

$$P(A) = \frac{\#\ of\ outcomes\ favourable\ to\ A}{total\ \#\ of\ outcomes\ in\ the\ sample\ space} = \frac{n(A)}{n(S)}$$

But just because it's the theoretical probability, doesn't mean the probability will occur that way.

For example, use the previous formula to determine the theoretical probability of flipping a coin, where "heads" is the favourable outcome.

According to our result, if we did an experiment where we flipped a coin 4 times, we should get 2 heads and 2 tails, but will this always be the case?

Of course not.

The **experimental probability** of event *A* occurring in multiple trials is given by:

$$P(A) = \frac{\#\ of\ times\ A\ actually\ occurs}{total\ \#\ of\ trials}$$

Experimental probability could be the same, or different than the theoretical probability.

Let's consider an example where we place 4 black chips and 5 white chips in a bag, and blindly pick one at random.

What is the theoretical probability of the following events:

P(black) = $\frac{4}{9}$ P(not black) = $\frac{5}{9}$

What is the relationship between these 2 events? What do their probabilities sum to?

P(black) and P(not black) are complementary events.

$$P(\text{black}) + P(\text{not black}) = \frac{4}{9} + \frac{5}{9} = \frac{9}{9} = 1$$

Probability is sometimes listed as a ratio called "odds."

Using the chip example above, P(B) was listed as 4/9

We can say the **odds for** selecting black are 4:5

Or, the **odds against** selecting black are 5:4

Ratios can be represented by fractions.

Odds are considered to be a *part : part* ratio, whereas probability is $\frac{part}{whole}$

Finally, even odds are when the odds for and against the event are equal, usually listed as 1:1, or 50:50.

Example

a) After suffering a severe heart attack, Dudley's doctor told him that he had even odds of making a full recovery. What is the probability that Dudley makes a full recovery?

$P(R) = \frac{\text{part}}{\text{whole}} = \frac{1}{2} = 0.5$

b) Stephanie has been told that the probability that she will make a full recovery is 0.65. Express, in lowest terms, the odds in favour of Stephanie making a full recovery.

$0.65 = \frac{65}{100} = \frac{13}{20}$

↳ out of a whole of 20 parts, 13 will lead to recovery.

odds part:part

13:7 } must add to the whole (20)

Example

When placing bets on horse races, the chances that each horse will win are often quoted as odds.
In a particular five-horse race, the odds against each horse winning are quoted as follows.

Horse	Free Spirit	Willow	Foxtrot	Gloria	Lucky Joe
Odds	2:1	3:1	7:2	8:1	11:1

→ odds against, so smallest ratio wins

a) Which horse is the favourite to win the race? Free Spirit.

b) Describe what is meant by the phrase "odds 2:1 against".

Free Spirit will lose 2 times for every one time he wins.

c) Determine the probabilities of the horses winning the race.

FS $\frac{1}{3}$ W $\frac{1}{4}$ F $\frac{2}{9}$ G $\frac{1}{9}$ LJ $\frac{1}{12}$

d) Determine the sum of the probabilities in c). Explain your answer.

$\frac{1}{3} + \frac{1}{4} + \frac{2}{9} + \frac{1}{9} + \frac{1}{12} = 1$ } one horse must win.

e) What are the odds in favour of "Foxtrot" winning the race?

Odds against: 7:2

Odds in favour: 2:7