3.3 - Mutually Exclusive Events

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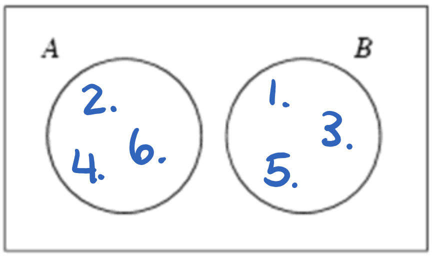
Remember from the last unit the intersection of 2 sets ($A \cap B$) as well as the union of 2 sets ($A \cup B$).

For this unit, instead of sets we're going to be discussing events, or rather elements from the sample set which are favourable to a given event.

If we have two separate events, event A and event B, the event ***A and B*** ($A \cap B$) occurs if both event A and event B happen simultaneously.

The one we're going to discuss today, ***A or B*** ($A \cup B$) occurs when only event A occurs, only when event B occurs, or if both events occur.

Consider the experiment of rolling a die and noting the result.
Let the event *A* be "an even number is thrown" and the event *B* be "an odd number is thrown".



a) Mark the outcomes of the experiment on the Venn diagram, which represents the sample space.

b) List the outcomes for:

i) the event ***A***

$\{2, 4, 6\}$

ii) the event ***B***

$\{1, 3, 5\}$

iii) the event $A \cup B$

$\{1, 2, 3, 4, 5, 6\}$

iv) the event $A \cap B$

$\{\emptyset\}$

c) Let $n(A)$ represent the number of outcomes in event A.

Complete the following:

$n(A) = 3$ $n(B) = 3$ $n(A \cup B) = 6$ $n(A \cap B) = 0$

d) Determine the following probabilities:

$P(A) = \frac{3}{6} = 1/2$ $P(B) = \frac{3}{6} = 1/2$ $P(A \cup B) = 6/6 = 1$ $P(A \cap B) = 0/6 = 0$

You'll notice that these sets are **disjoint**, that is they have no overlap.

★ When this happens, we say that events A and B are mutually exclusive.

Mutually exclusive events have no outcomes that are common.

Are the following statements true for the previous example?

$n(A \cup B) = n(A) + n(B)$

$6 = 3 + 3$

$6 = 6$

✓ True.

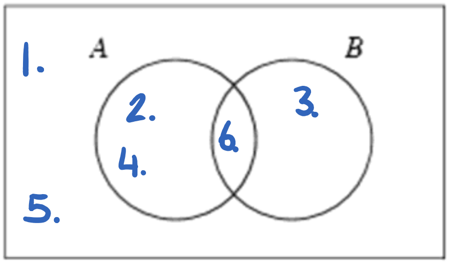
$P(A \cup B) = P(A) + P(B)$

$1 = \frac{1}{2} + \frac{1}{2}$

$1 = 1$

✓ True.

Consider the experiment of rolling a die and noting the result.
Let the event A be "an even number is thrown" and the event B be "a multiple of three is thrown".



a) Mark the outcomes of the experiment on the Venn diagram.

b) List the outcomes for:

i) the event A

$\{2, 4, 6\}$

ii) the event B

$\{3, 6\}$

iii) the event $A \cup B$

$\{2, 3, 4, 6\}$

iv) the event $A \cap B$

$\{6\}$

c) Complete the following:

$n(A) =$ 3 $n(B) =$ 2 $n(A \cup B) =$ 4 $n(A \cap B) =$ 1

d) Determine the following probabilities:

$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{2}{6} = \frac{1}{3}$ $P(A \cup B) = 4/6 = \frac{2}{3}$ $P(A \cap B) = \frac{1}{6}$

In this example, however, the events A and B do have a common outcome(s).

So in this case, event A and event B are **not** mutually exclusive.

If we discuss the outcomes with the above Venn diagram, you'll notice that the sets are no longer disjoint. That is, they have some overlap which we should recognize as the set $(A \cap B)$.

From the above example, verify the following two equations:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$4 = 3 + 2 - 1$

$4 = 4$ ✓ True!

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{2}{3} = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

$\frac{2}{3} = \frac{2}{3}$ ✓ True.

In both of these equations, because there is an area of overlap ($(A \cap B)$), we counted that area of overlap **two times**, once when we count set A, and once more when we count set B. Because we only want to count it once when determining $n(A \cup B)$ or $P(A \cup B)$, we have to subtract **only one** of those double-counts after we added all of B and all of A. The doubly-counted part is the overlap, otherwise known as the intersection. That's why we see "$-n(A \cap B)$" or "$-P(A \cap B)$" at the end of those equations.

The big ideas for this chapter are:

- Events are **mutually exclusive** if they share no common outcomes. **Complementary events** will always be mutually exclusive.
- Two events, A and B, are mutually exclusive if $P(A \cap B) = 0$.
- Use the following formulas to determine the probability of the event $A \cup B$:

Do not share outcomes

If event A and B are mutually exclusive:

$$\boldsymbol{P(A \cup B) = P(A) + P(B)}$$

Share outcomes

If event A and B are **not** mutually exclusive:

$$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Example

In each case, state whether the events A, B are mutually exclusive or not.

a) Experiment - a die is rolled

Event A - an even number occurs Event B - an odd number occurs

mutually exclusive.

b) Experiment - a card is drawn from a standard deck

Event A - a face card is selected Event B - a club is selected

not mutually exclusive, ie. King of clubs.

c) Experiment - two dice are thrown

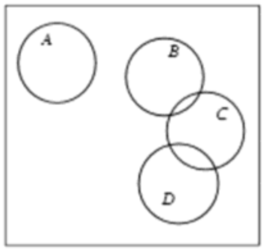
Event A - the dice both show the same value Event B - the total score is 11

mutually exclusive.

Example

Events have been represented on a Venn diagram.
State all the pairs of events that are mutually exclusive.

A & B, A & C, A & D, B & D



Example

Use the given information and the probability formula to determine whether the events A, B are mutually exclusive.

$$P(A) = \frac{1}{4} \qquad P(B) = \frac{1}{3} \qquad P(A \cup B) = \frac{7}{12}$$

Mutually exclusive if:

$$P(A \cup B) = P(A) + P(B)$$

$$\left(\frac{7}{12}\right) = \left(\frac{1}{4}{}^{\times 3}_{\times 3}\right) + \left(\frac{1}{3}{}^{\times 4}_{\times 4}\right)$$

$$\frac{7}{12} = \frac{3}{12} + \frac{4}{12}$$

$$\frac{7}{12} = \frac{7}{12} \quad \text{True!}$$

$\therefore$ Event A and B are mutually exclusive

Example

A card is drawn from a standard deck of 52 cards. Use formulas to determine the probability that

a) a nine of diamonds or a heart is drawn

Event A: 9♢
Event B: ♡

mutually exclusive!

$P(A \cup B) = P(9♢) + P(♡)$

$= \frac{1}{52} + \frac{13}{52}$

$= \frac{14}{52} = \boxed{\frac{7}{26}}$

b) a nine or a heart is drawn

A: 9
B: ♡

not mutually exclusive
ie. 9 of hearts.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$

$= \frac{4}{13}$

Example

Blocks of different shapes and colours are placed into a bag.
There are three cubes: one blue, one red, and one green.
There are two pyramids: one red and one yellow.
There are five cylinders: one blue, one yellow and three red.

Determine the probability that a block selected at random

a) will be a pyramid or a cube

mutually exclusive

$P(P \cup C_u) = P(P) + P(C_u)$

$= \frac{2}{10} + \frac{3}{10} = \frac{5}{10}$

$= 1/2$

b) will not be red

5 red
5 not red

$\frac{5}{10} = \frac{1}{2}$

c) will be red or a cylinder

not mutually exclusive,
ie. red cylinders.

$P(r \cup C_y) = P(r) + P(C_y) - P(r \cap C_y)$

$= \frac{5}{10} + \frac{5}{10} - \frac{3}{10} = \frac{7}{10}$