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FOUNDATIONS OF MATH 12 Ch. 3 – Day 4: Independent/Dependent Events and Event "$A \cap B$"

Let's consider two similar, but different experiments.

Firstly, we'll take a normal deck of cards and draw one card. We'll then take that card and place it back in the deck, shuffle it, and then draw another card.

Let's consider the following events, $A = \{the\ first\ card\ is\ a\ heart\}$, and $B = \{the\ second\ card\ is\ a\ heart\}$.

Determine the following:

$P(A) = \frac{13}{52} = \frac{1}{4}$ $P(B) = \frac{13}{52} = \frac{1}{4}$

In this case, the probability of event B does not depend on whether event A occurred.

We say the event A and B are Independent.

Secondly, we'll do the exact experiment, but we won't replace the first card. We'll leave it out of the deck while we draw the second card.

Let's consider the following events, $A = \{the\ first\ card\ is\ a\ heart\}$, and $B = \{the\ second\ card\ is\ a\ heart\}$.

Determine the following:

$P(A) = \frac{13}{52} = \frac{1}{4}$

But how can we determine P(B)? It's impossible, why?

It depends on whether the first card was a heart.

In this case, the probability of event B occurring depends on whether event A occurred.
Event A and B are said to be dependent events.

Example

Classify the following events as dependent or independent.

a) The experiment is rolling a die and tossing a coin.
The first event is rolling 2 on the die and the second event is tossing tails on the coin.

Independent

b) The experiment is choosing two cards without replacement from a standard deck.
The first event is that the first card is a king and the second event is that the second card is a king.

Dependent

c) The experiment is choosing two cards with replacement from a standard deck.
The first event is that the first card is a king and the second event is that the second card is a king.

Independent.

Let's talk some more about the second experiment from the beginning of the section. As we determined, the probability of the first card being a heart is one fourth, but event B depends whether event A did or did not occur. This is known as conditional probability.

We denote conditional probability as follows: $P(B|A)$. This describes the probability of event B occurring if event A **did** occur.

Clearly, the other option is the probability of event B happening if event A **did not** occur. That would be denoted as follows: $P(B|A')$.

In our example, determine:

$P(B|A) = \frac{12}{51}$

$P(B|A') = \frac{13}{51}$

Now, let's discuss the event $A \cap B$. Because it's the intersection of two (or more) events, the events will either be independent or dependent, and the probability changes accordingly. Let's take a look at an example to explore what the probability will be.

We'll take a red die and a blue die and roll them. All the outcomes (the sample space) are shown in the following table:

		Blue					
		1	2	3	4	5	6
Red	1	{1,1}	{1,2}	{1,3}	{1,4}	{1,5}	{1,6}
	2	{2,1}	{2,2}	{2,3}	{2,4}	{2,5}	{2,6}
	3	{3,1}	{3,2}	{3,3}	{3,4}	{3,5}	{3,6}
	4	{4,1}	{4,2}	{4,3}	{4,4}	{4,5}	{4,6}
	5	{5,1}	{5,2}	{5,3}	{5,4}	{5,5}	{5,6}
	6	{6,1}	{6,2}	{6,3}	{6,4}	{6,5}	{6,6}

We'll consider two events:

$A = \{the\ sum\ of\ the\ two\ die\ is\ 10\}$, $B = \{the\ number\ on\ each\ die\ is\ the\ same\}$

List the outcomes that are favourable to each event:

$A = \{(4,6), (5,5), (6,4)\}$

$B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Determine the following:

$P(A) = \frac{3}{36} = \frac{1}{12}$ $P(B) = \frac{6}{36} = \frac{1}{6}$

$P(B|A) = \frac{1}{3}$ $P(A|B) = \frac{1}{6}$

$P(A \cap B) = \frac{1}{36}$ $P(B \cap A) = \frac{1}{36}$

Now let's verify the following:

$P(A \cap B) = P(A) \times P(B|A)$

$\frac{1}{36} = \frac{1}{12} \times \frac{1}{3}$

$\frac{1}{36} = \frac{1}{36}$ ✓

$P(B \cap A) = P(B) \times P(A|B)$

$\frac{1}{36} = \frac{1}{6} \times \frac{1}{6}$

$1/36 = 1/36$ ✓

$P(A \cap B) = P(B \cap A)$

$\frac{1}{36} = \frac{1}{36}$

✓

Think of a Venn diagram, does it make sense that $P(A \cap B) = P(B \cap A)$?

Yes, they're both the same
overlapping area.

So, in general:

If the events A and B are **dependent:**
$$P(A \cap B) = P(A) \times P(B|A)$$

If the events A and B are **independent:**
$$P(A \cap B) = P(A) \times P(B)$$

Note: Independent events are not the same as mutually exclusive events. Mutually exclusive events describe whether or not two events can occur at the same time, while independent events describe if one event has an effect on the probability of another event.

Example

dependent

Two cards are drawn **without** replacement from a standard deck of 52 cards. Determine the probability of the following events.

a) Both cards are red.

$P(r) \times P(r|r)$

$= \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$

b) Neither card is a club.

$P(c') \times P(c'|c')$

$= \frac{39}{52} \times \frac{38}{51} = \frac{19}{34}$

c) The first card is a king and the second card is a five.

$P(K) \times P(5|K)$

$= \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

d) One of the cards is a king and the other is a five.

K,5 or 5,K

$P(K) \times P(5|K) + P(5) \times P(K|5)$

$= \frac{4}{52} \times \frac{4}{51} + \frac{4}{52} \times \frac{4}{51} = \frac{8}{663}$

Example

independent

Two cards are drawn **with** replacement from a standard deck of 52 cards. Determine the probability of the following events.

a) Both cards are red.

$\frac{26}{52} \times \frac{26}{52} = \frac{1}{4}$

b) The first card is a king and, the second card is a five.

$P(K) \times P(5) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

c) One of the cards is a king and the other is a five.

K,5 or 5K.

$P(K) \times P(5) + P(5) \times P(K)$

$\frac{4}{52} \times \frac{4}{52} + \frac{4}{52} \times \frac{4}{52} = \frac{2}{169}$

Example

If $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{5}$, and $P(A \cup B) = \frac{3}{5}$, use appropriate formulas to investigate whether event A and event B are

a) mutually exclusive events

$P(A) + P(B)$

$\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$

$= \frac{11}{15}$

Since $P(A) + P(B) \neq P(A \cup B)$, A & B are not mutually exclusive.

b) independent events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 1/3 + 2/5 - 3/5$

$= 2/15$

For independent, $P(A \cap B) = P(A) \cdot P(B)$

$P(A) \cdot P(B) = 1/3 \cdot \frac{2}{5} = \frac{2}{15}$

They are both equal, so independent.

Example

Let A and B be events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$.

Determine:

a) $P(B|A)$

$P(A \cap B) = P(A) \cdot P(B|A)$

$\frac{1}{4} = \frac{1}{2} \cdot P(B|A)$

$\Rightarrow P(B|A) = \frac{1}{4} \div \frac{1}{2}$

$= \frac{2}{4} = \frac{1}{2}$

b) $P(A|B)$

$P(B \cap A) = P(B) \cdot P(A|B)$

$\frac{1}{4} = \frac{1}{3} \cdot P(A|B)$

$\Rightarrow P(A|B) = \frac{1}{4} \div \frac{1}{3}$

$= \frac{3}{4}$

c) $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 1/2 + 1/3 - 1/4$

$= \frac{6}{12} + \frac{4}{12} - \frac{3}{12}$

$= 7/12$

Example

The probability that Sophia will pass Math this semester is 0.7 and the probability that she will pass English this semester is 0.9. If these events are independent, determine the probability (to the nearest hundredth) that she will pass

a) Math and English

$P(M \cap E) = P(M) \cdot P(E)$

$= 0.7 \times 0.9 = 0.63$

b) Math or English

$P(M \cup E) = P(M) + P(E) - P(M \cap E)$

$= 0.7 + 0.9 - 0.63$

$= 0.97$

c) Math but not English

$P(M \cap E') = P(M) \cdot P(E')$

$= 0.7 \cdot 0.1$

$= 0.07$

d) neither Math nor English

$P(M' \cap E') = P(M') \cdot P(E')$

$= 0.3 \cdot 0.1$

$= 0.03$