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Recall the four formulas we've explored so far:

If events A and B are mutually exclusive,

$$P(A \cup B) = P(A) + P(B)$$

If events A and B are NOT mutually exclusive,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If events A and B are independent,

$$P(A \cap B) = P(A) \times P(B)$$

If events A and B are dependent,

$$P(A \cap B) = P(A) \times P(B|A)$$

Let's explore an example with a table:

The table shows how the students in a large high school generally travel to school.

	Bus B	Car C	Other O	Total
Male, M	350	200	75	625
Female, F	300	175	100	575
Total	650	375	175	1200

a) Complete the totals in the chart.

b) How many students attend the high school? 1200

c) Use the numbers in the table to determine

i) $P(M)$ $= \frac{625}{1200} = \frac{25}{48}$

ii) $P(C|M)$ $= \frac{200}{625} = \frac{8}{25}$

iii) $P(M|C)$ $= \frac{200}{375} = \frac{8}{15}$

iv) $P(M \cap C)$ $= \frac{200}{1200} = 1/6$

d) If a student is selected at random, determine the probability that the student

i) is female $\frac{575}{1200} = \frac{23}{48}$

ii) travels by bus $\frac{650}{1200} = \frac{13}{24}$

iii) is female and travels by bus $\frac{300}{1200} = \frac{8}{15}$

e) Identify the following events and determine the probability that

i) a female student travels by bus

$P(B|F) = \frac{300}{575}$

ii) a student who drives to school is male

$P(m|C) = \frac{200}{375} = \frac{8}{15}$

f) Are the events "the student is female" and "the student travels by bus" independent events? Explain.

$P(F \cap B) = \frac{1}{4}$, $P(F) \cdot P(B) = \frac{23}{48} \cdot \frac{13}{24} = \frac{299}{1152}$

not equal, $\therefore$ not independent.

Sometimes using a tree diagram can be helpful to visualize outcomes:

Students were given the following problem to solve:

"The odds that Ava will pass Math this semester are 4:1 in favour and the odds that she will pass English this semester are 1:9 against. If these events are independent, determine the probability (to the nearest hundredth) that she will pass

i) Math and English **ii)** Math and not English **iii)** Math or English"

a) Express the odds in terms of probabilities.

$P(M) = 4/5 \qquad P(E) = 9/10$

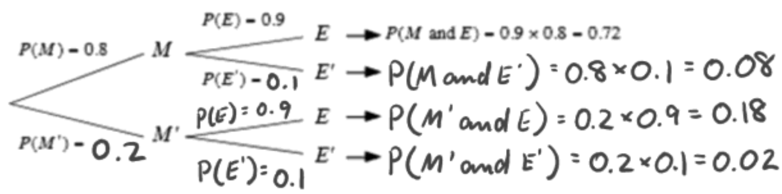
b) Some of the students chose to use probability formulas to solve the problem. Show this method of solution below.

i) $P(M \cap E) = P(M) \cdot P(E) = \frac{4}{5} \cdot \frac{9}{10} = \frac{18}{25}$

ii) $P(M \cap E') = P(M) \cdot P(E') = \frac{4}{5} \cdot \frac{1}{10} = \frac{2}{25}$

iii) $P(M \cup E) = P(M) + P(E) - P(M \cap E) = \frac{4}{5} + \frac{9}{10} - \frac{18}{25} = \frac{49}{50}$

c) Other students chose to use a probability tree diagram to solve the problem. Part of the tree diagram is shown. Complete the tree diagram and determine the solution to the problem.



i) 0.72 ii) 0.08 iii) $0.72 + 0.08 + 0.18 = 0.98$

This is an example of a problem solved with a tree diagram where the events were independent.

Let's take a look at how to set one up for two events that are dependent.

Cheryl is trying to show Jon how to solve problems based on the following information.

" Two machines, A_1 and A_2, produce all the glass bottles made in a factory. The percentages of broken bottles produced by these machines are 5% and 8% respectively. Machine A_1 produces 60% of the output. "

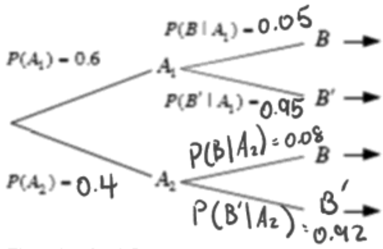
Cheryl suggests the following strategy.

First: Introduce symbols to represent the information.
Second: Write the given probabilities in terms of the symbols.
Third: Set up a probability tree diagram.

a) Complete Jon's work, which is started below.

- A_1 - bottle is from machine A_1. A_2 - bottle is from machine A_2. B - bottle is broken.
- $P(A_1) = 0.6$ $P(A_2) = 0.4$ $P(B|A_1) = 0.05$ $P(B|A_2) = 0.08$

b) Jon sets up a probability tree diagram, with the first branches leading to the machines and the second set of branches leading to the broken/not broken items.
Complete the diagram.



$B \rightarrow P(A_1 \cap B) = P(A_1)\,P(B | A_1) = 0.6 \times 0.05 = 0.03$

$B' \rightarrow P(A_1 \cap B') = P(A_1)P(B'|A_1) = 0.6 \times 0.95 = 0.57$

$B \rightarrow P(A_2 \cap B) = P(A_2)P(B|A_2) = 0.4 \times 0.08 = 0.032$

$B' \rightarrow P(A_2 \cap B') = P(A_2)P(B'|A_2) = 0.4 \times 0.92 = 0.368$

c) Cheryl asked Jon some questions to see if he understands what he has drawn.

i) If a bottle is chosen at random, determine the probability that it is a broken bottle produced by machine A_1.

$P(A_1 \cap B) = 0.03$

ii) Which of the final outcomes in the diagram relate to the event "a bottle is not broken"?

$A_1 \cap B'$, $A_2 \cap B'$

iii) If a bottle is chosen at random, determine the probability that the bottle is not broken.

$P(A_1 \cap B') + P(A_2 \cap B')$

$= 0.57 + 0.368$

$= 0.938 = 93.8\%$

As part of an experiment into the learning process, a mouse is put into a maze and rewarded with food every time it turns right. If the mouse turns right, the probability that it turns right the next time is increased by 20%. If the mouse turns left, the probability that it turns left the next time is decreased by 20%. Assume that there is an equal probability that the first turn will be to the left or to the right.

Let R_1 and L_1 represent the events "the first turn is to the right" and "the first turn is to the left" respectively.

a) What is meant by the event $R_2 | R_1$?

The mouse turned right the second time, given it turned right the first time.

b) Determine, as an exact decimal, the probability of each event.

i) The first turn is to the right.

$P(R_1) = \frac{1}{2}$

ii) The first turn is to the left.

$P(L_1) = \frac{1}{2}$

iii) The second turn is to the right, given that the first turn is to the right.

$$P(R_2|R_1) = 0.5 + 20\% \text{ of } 0.5$$
$$= 0.5 + 0.1 = 0.6$$

iv) The second turn is to the left, given ~~that the~~ first turn is to the left.

$$P(L_2|L_1) = 0.5 - 20\% \text{ of } 0.5$$
$$0.5 - 0.1 = 0.4$$

c) Put the probabilities in b) onto a probability tree diagram.
Complete the diagram and use it to determine the probability that the first two turns are

i) both to the right **ii)** both to the left **iii)** different

0.5 R_1 — 0.6 $R_2 \rightarrow P(R_1 \cap R_2) = 0.5 \cdot 0.6 = 0.3$

0.5 R_1 — 0.4 $L_2 \rightarrow P(R_1 \cap L_2) = 0.5 \cdot 0.4 = 0.2$

0.5 L_1 — 0.6 $R_2 \rightarrow P(L_1 \cap R_2) = 0.5 \cdot 0.6 = 0.3$

0.5 L_1 — 0.4 $L_2 \rightarrow P(L_1 \cap L_2) = 0.5 \cdot 0.4 = 0.2$

i) 0.3 ii) 0.2 iii) 0.2+0.3 = 0.5