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FOUNDATIONS OF MATH 12 Ch. 3 – Day 6: Probability Involving Permutations & Combinations

As you can imagine, the concepts we've learned from our permutations and combinations unit can help us determine the sample space of more complicated problems.

Let's review some formulas from that section:

Factorial notation: $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1, \textit{where } n \in \mathbb{W}$

Permutations (order matters): ${}_nP_r = \frac{n!}{(n-r)!}$

Permutations with repetitions: $\frac{n!}{a!b!c!}$

Combinations (order doesn't matter): ${}_nC_r = \frac{n!}{(n-r)!r!}$

Example

Two cards are selected without replacement from a deck of 52 playing cards. Determine the probability that both cards are kings using

a) the multiplication law **b)** combinations

$$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \frac{1}{221}$$

acceptable outcomes

$$\frac{{}_4C_2}{{}_{52}C_2} = \frac{1}{221}$$

total outcomes

Example

In a card game, you are dealt 5 cards from a pack of 52 shuffled cards.
When you look at your 5 cards, what is the probability, expressed in combination notation, that you have

a) four aces?

4 aces and 1 other card:

$$\frac{{}_4C_4 \times {}_{48}C_1}{{}_{52}C_5}$$

b) four tens and an ace?

$$\frac{{}_4C_4 \times {}_4C_1}{{}_{52}C_5}$$

c) 10, J, Q, K, and ace?

$$\frac{{}_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1 \times {}_4C_1}{{}_{52}C_5}$$

$$= \frac{({}_4C_1)^5}{{}_{52}C_5}$$

d) at least one Jack?

complement

$$1 - P(\text{no jacks})$$

$$1 - \left(\frac{{}_{48}C_5}{{}_{52}C_5}\right)$$

Example

The word **COUNTED** has been spelled using Scrabble tiles. Two tiles are randomly chosen one at a time and placed in the order in which they were chosen. Determine the probability that the tiles are

a) **CO**

Permutations!

$$P(CO) = \frac{{}_2P_2}{{}_7P_2} = \frac{1}{42}$$

b) both vowels

$$\frac{{}_3P_2}{{}_7P_2} = \frac{6}{42} = \frac{1}{7}$$

Example

Mike's MP3 player contains 50 songs. If he listens to 10 songs on "random", determine the probability, as a percentage to the nearest hundredth, that the playlist contains

a) Mike's two favourite songs

$$\frac{{}_{2}C_{2} \times {}_{48}C_{8}}{{}_{50}C_{10}} = 0.03673$$

b) one of his two favourite songs at the beginning of the 10 songs and the other at the end

order matters.

② (8, any order) ①

$$\frac{{}_{2}P_{2} \times {}_{48}P_{8}}{{}_{50}C_{10}} \doteq 0.000816\ldots$$

$$= 0.08\%$$

Example

City Council consists of nine men and six women. Three representatives are chosen at random to form an environmental sub-committee.

a) What is the probability that Mayor Jim Milonovich and two women are chosen?

$$\frac{{}_{1}C_{1} \times {}_{6}C_{2}}{{}_{15}C_{3}} = \frac{3}{91}$$

b) What is the probability that two women are chosen if Mayor Jim Milonovich must be on the committee?

Mayor is locked in.

⇒ Pick 2 women from 14 people.

$$\frac{{}_{6}C_{2}}{{}_{14}C_{2}} = \frac{15}{91}$$

Example

A hacker is attempting to break into a friend's security-protected file. The friend tells the hacker all the numbers that are in the 4-digit PIN but not the order or how many times each digit may be repeated in the PIN.

Determine the probability that the hacker correctly guesses the 4-digit PIN on the first attempt if the friend tells her that the PIN contains

a) the numbers 4, 5, 6, and 7 $4! = 24$ arrangements. $\frac{1}{24}$ chance.

b) only the numbers 4, 5, and 6

4, 4, 5, 6 in any order : $4!/2! = 12$
4, 5, 5, 6 in any order : $4!/2! = 12$
4, 5, 6, 6 in any order : $4!/2! = 12$

36 possibilities. $1/36$ chance.

c) only the numbers 4 and 5

4, 4, 4, 5 in any order ; $4!/3! = 4$
4, 4, 5, 5 in any order ; $4!/2! = 4$
4, 5, 5, 5 in any order ; $4!/3! = 6$

14 possibilities $1/14$

d) only the number 4

4, 4, 4, 4 : 1 arrangement.

$1/1 = 1$