1.2 - Set Theory

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In math, a "set" is any collection filled with objects. For example, the numbers on the faces of a die can form a set.

Objects that are found within the set are called "elements." For example, the number "5" would be an **element** within the **set** of "numbers on a die."

Usually we represent sets with capital letters. We can call the above set "A" and list its elements in one of 3 ways within a set of braces: {}

- We can list them: $A = \{1, 2, 3, 4, 5, 6\}$
- We can describe them in words: $A = \{natural\ numbers\ less\ than\ 7\}$
- We can use set builder notation: $A = \{x \mid x < 7, x \in \mathbb{N}\}$

For set builder notation, the bracers together are read as "the set of."
"|" begins the description of the element(s) in the set and is read as "such that." The description of the elements may contain several descriptors separated by commas.

The number of elements in a set are written as $n(A)$. In the die example, n(A) = 6.

$\in$ represents, "is an element of," and $\notin$ represents, "is not an element of."

Using the above example, the number 2 belongs to set A, but the number 8 does not. To describe this, we can write $2 \in A$ and $8 \notin A$.

Also, the order that the elements are listed in doesn't matter, ie. {1,2} = {2,1}

Finally, recall the number systems are their symbols:

Natural numbers, $\mathbb{N} = 1, 2, 3, \ldots$
Whole numbers, $\mathbb{W} = 0, 1, 2, \ldots$
Integers, $\mathbb{Z} = \ldots -2, -1, 0, 1, 2, \ldots$
Rational numbers, $\mathbb{Q} = any\ number\ that\ can\ be\ written\ as\ a\ fraction, \frac{a}{b}$
Irrational numbers, $\overline{\mathbb{Q}} = any\ number\ that\ can't\ be\ written\ as\ a\ fraction, \pi, \sqrt{2}$
Real numbers, $\mathbb{R} = all\ rational\ and\ irrational\ numbers$

Example

Consider the following 2 sets:

$$P = \{whole\ numbers\ less\ than\ or\ equal\ to\ 3\}$$
$$Q = \{even\ whole\ numbers\ less\ than\ 10\}$$

$\leq$

List the elements of the sets:

$P = \{0, 1, 2, 3\}$

$Q = \{0, 2, 4, 6, 8\}$

Complete the following:

$n(P) =$ 4

$n(Q) =$ 5

Write set P using set builder notation:

$P = \{x \mid x \leq 3, x \in \mathbb{W}\}$

Which of the following represents set Q?

~~A.~~ $Q = \{x|x < 10, x \in \mathbb{W}\}$

~~B.~~ $Q = \{e|e = 2x, 0 \leq x \leq 10, x \in \mathbb{W}\}$

C. $Q = \{e|e = 2x, 0 \leq x \leq 4, x \in \mathbb{W}\}$

$e = 2x$

$e = 2(0) = 0$

$e = 2(1) = 2$

$e = 2(2) = 4$

$e = 2(3) = 6$

$e = 2(4) = 8$

D. $Q = \{e|e = 2x, 0 \leq x \leq 5, x \in \mathbb{W}\}$

$e = 2x$

$2(0) = 0$

$2(1) = 2$

$2(2) = 4$

$2(3) = 6$

$2(4) = 8$

$2(5) = 10$

Next, we're going to talk about a few special sets. To discuss them, lets define a few other sets so we can use them as an example.

These sets will contain **non-negative single digit** numbers. List the elements of the sets:

$E = \{even\ numbers\}$ $E = \{0, 2, 4, 6, 8\}$
$L = \{whole\ numbers\ less\ than\ 7\}$ $L = \{0, 1, 2, 3, 4, 5, 6\}$
$O = \{odd\ numbers\ less\ than\ 7\}$ $O = \{1, 3, 5\}$

The Universal set is the set which contains **all** elements we are discussing. It will most likely be different from problem to problem; it depends what elements are currently being discussed. It is always the largest set.

What is the universal set for the above example? Label it "*U*."

U = {non-negative, single-digit numbers}

A Subset of a set is a set than contains none, some, or all the elements from a preexisting set. Because the universal set is the largest set, all sets will be a subset of the universal set, and every set is considered to be a subset of itself.

In set notation, the symbol ⊂ is used to identify subets.

For example, set *E* is a subset of the universal set *U*, so we can write "$E \subset U$". This is read as "set E is a subset of set U."

If a set is not a subset of a predefined set, the symbol ⊄ represents "not a subset of."

Is *E* a subset of *L*? describe it using appropriate set symbols.

$E \not\subset L$

Set *O* is a subset of set *L* which is a subset of set *U*. Write this relationship.

$O \subset L \subset U$

Our last special set is the empty.

The empty set is just that; it's empty. It contains no elements. Because it contains no elements, it is a subset of all sets. It's given the symbol ∅.

For example, the set of odd numbers in set *E* is ∅ because there are no odd numbers in set *E*.

Remember the complement from the last unit? We used it in some "at least" and "at most" problems. The complement was what was left over from our unrestricted answer after we took out what we were trying to determine.

In set theory, it's very similar. The complement of set *A* is the set of elements in the universal set that are not in set A. The complement of set *A* is denoted by *A′* ("A-prime").

$A' = U - A$, where *U* is the universal set.

Example

Consider the following two sets:

$A = \{natural\ numbers\ less\ than\ 20\ which\ are\ divisible\ by\ 3\}$
$B = \{natural\ numbers\ less\ than\ 20\ which\ are\ divisible\ by\ 5\}$

Why is the set of natural numbers less than 20 a suitable universal set in this case?

B/c all the elements we're discussing are in that universal set.

List the elements of the following sets:

$A = \{3, 6, 9, 12, 15, 18\}$ $B = \{5, 10, 15\}$

$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19\}$

Write a description of the elements of the complement of set *B* in words.

Natural numbers less than 20
that are not divisible by 5.

Let's define set $C = \{natural\ numbers\ less\ than\ 20\ that\ are\ multiples\ of\ 6\}$.

$C = \{6, 12, 18\}$

State whether the following are true or false:

i) $C \subset A$?

A contains all
elements from set C,
$\therefore C \subset A$
is true.

ii) $C \not\subset B$

B contains
no elements
from set C,
$\therefore C \not\subset B$
is true.

iii) $A' \subset C'$

C' contains all
elements from A',
$\therefore A' \subset C'$
is true.