1.3 - Venn Diagrams Pt. I

October 18, 2019 9:36 AM

Today we're going to begin to explore Venn diagrams and their usefulness in discussing set theory.

Let's use two sets from a previous example to help construct one:

$A = \{natural\ numbers\ less\ than\ 20\ that\ are\ divisible\ by\ 3\}$
$B = \{natural\ numbers\ less\ than\ 20\ that\ are\ divisible\ by\ 5\}$

Let's use appropriate notation to list the set of whole numbers less than 20 that are:

i) Divisible by 3:

$A = \{3, 6, 9, 12, 15, 18\}$

ii) Divisible by 5:

$B = \{5, 10, 15\}$

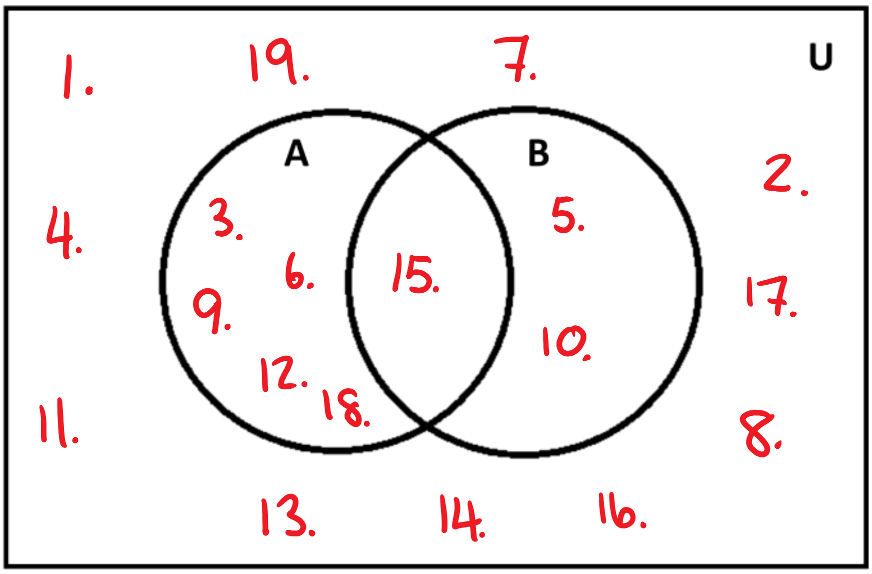
iii) Divisible by 3 and 5:

$A \cap B = \{15\}$

iv) Divisible by 3 or 5:

$A \cup B = \{3, 5, 6, 9, 10, 12, 15, 18\}$

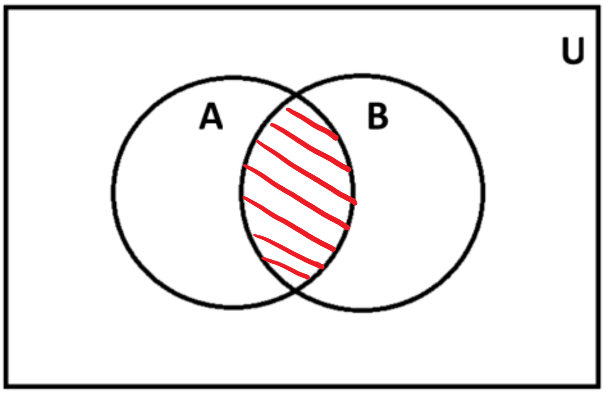
Now let's construct a Venn diagram to represent these sets. Notice how the universal set U is placed outside the circles to indicate the universal set contains all the elements. Note: Place dots after the elements in the set to indicate that it's an element.



Notice how *A* **and** *B* are located in the intersection of the circles, *A* **or** *B* is located in their own circle, and **neither** *A* or *B* is located outside the circles, but within the rectangle.

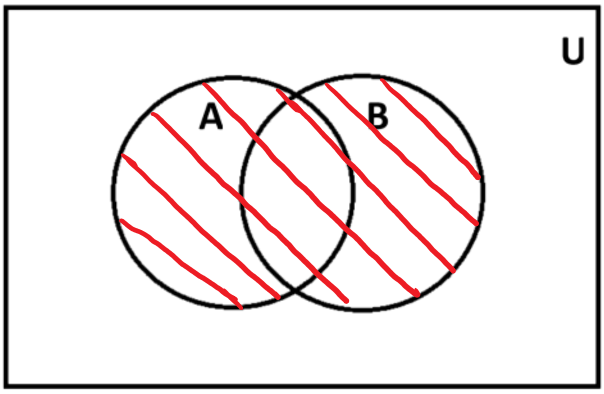
Example

Shade the intersection of set *A* and set *B* (ie. set *A* and *B*) $A \cap B$



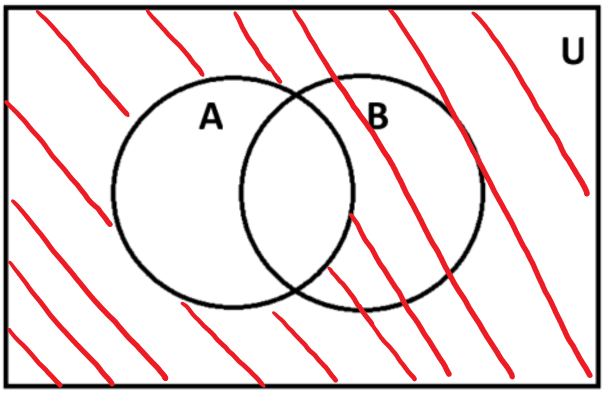
Example

Shade the union of set *A* and set *B*. (ie. set *A* or set *B*) $A \cup B$



Example

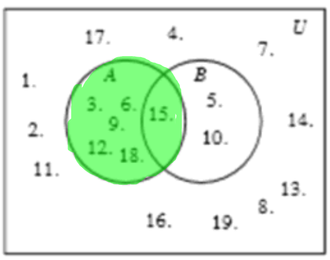
Shade the complement of set *A* (ie. the set not *A*)



Example

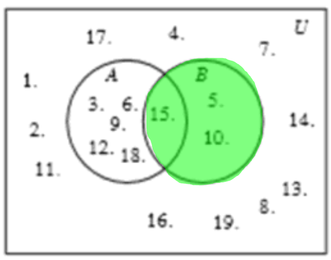
Let's use the two sets from the start of this section. Shade the region representing the given set, and also describe each set in terms of *A* and/or *B* and determine the number of elements in each set.

Shade set *A*



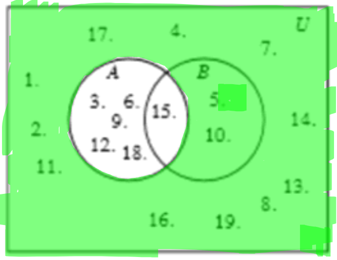
- A
- $n(A) = 6$

Shade set *B*



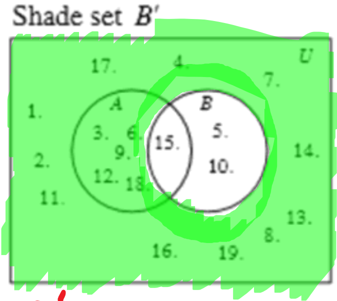
- B
- $n(B) = 3$

Shade set A'



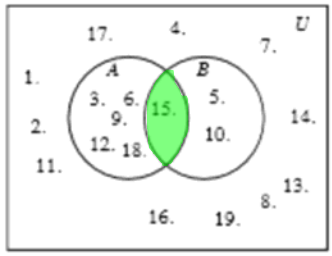
- A'
- $n(A') = 13$

Shade set B'



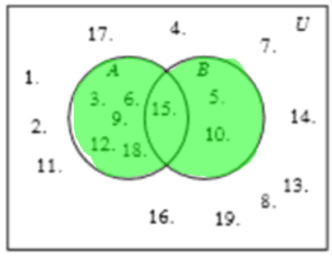
- B'
- $n(B') = 16$

Shade set $A \cap B$



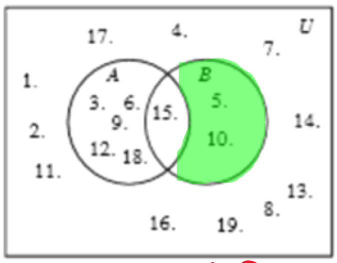
- A and B
- $n(A \cap B) = 1$

Shade set $A \cup B$



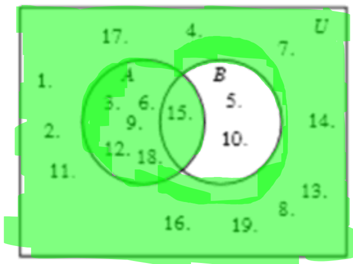
- $A \cup B$
- $n(A \cup B) = 8$

Shade set $A' \cap B$



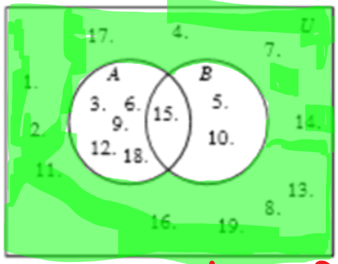
- not A and B
- $n(A' \cap B) = 2$

Shade set $A \cup B'$



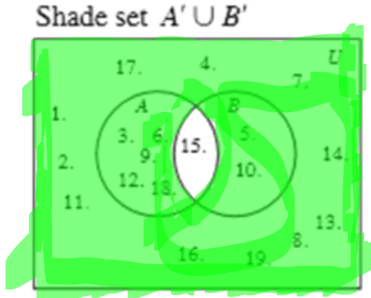
- A or not B
- $n(A \cup B') = 17$

Shade set $A' \cap B'$



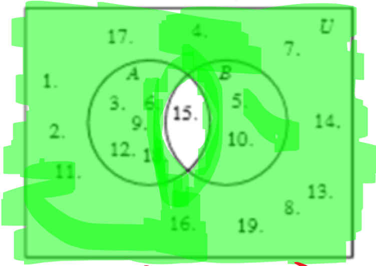
- not A and not B
- $n(A' \cap B') = 11$

Shade set $A' \cup B'$



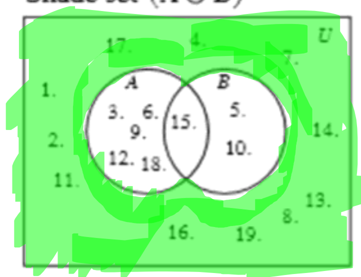
- not A or not B
- $n(A' \cup B') = 18$

Shade set $(A \cap B)'$



- not (A and B)
- $n((A \cap B)') = 18$

Shade set $(A \cup B)'$



- not (A or B)
- $n((A \cup B)') = 11$

From these diagrams we should have noticed two properties of set theory:

$$(A \cap B)' = A' \cup B'$$

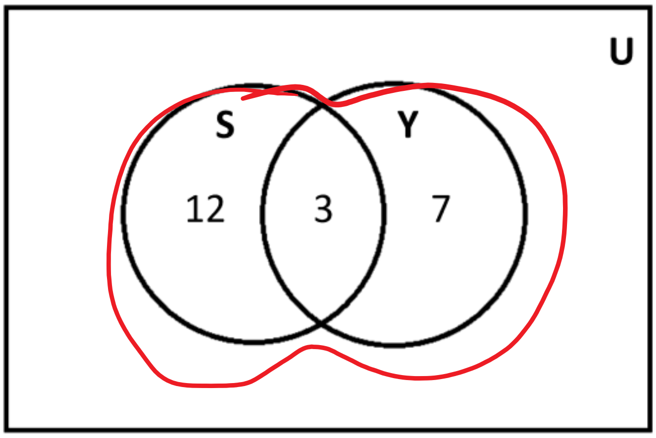
and also,

$$(A \cup B)' = A' \cap B'$$

We can also use Venn diagrams to represent the number of elements in each set. When we do this, do **not** place a dot after the entry.

Example

The following Venn diagram represents the number of students who are members of student council (set *S*) and the number of students who are part of yearbook (set *Y*):



How many students are on student council?

$12+3=15$

How many students are on student council and yearbook? What set is that?

$n(S \cap Y)=3$

How many are on year book, but not student council? What set is that?

$n(S')=7$

How many are on student council or yearbook? What set is that?

$n(S \cup Y)=22$