

1.5 - Conditional Statements

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A conditional statement always comes with some sort of condition (duh). It's given in the form "if <hypothesis>, then <conclusion>".

A conditional statement, also sometimes called an "if – then proposition," may be true or false.

For example, "if a Canadian citizen is 18 or older, then they are eligible to vote," and, "if a person lives in Vancouver, then they were born in BC," are both conditional statements, although one is true and one is false.

A conditional statement will also have a converse.

The converse of a conditional statement occurs when the hypothesis and the conclusion are swapped.

Example

Write the following statement as a conditional statement, and then write its converse.

"An acute triangle has three angles that are each less than 90° "

"^{hypothesis} If a triangle is acute, ^{conclusion} then it has 3 angles less than 90° ."

"If a triangle has 3 angles that are all less than 90° , then it is acute."

Depending on the statement, its converse may or may not have the same truth value of its converse.

We get a special kind of conditional statement when the conditional statement is true, as well as its converse. This is called a "biconditional" statement.

Both versions of a biconditional statement can be combined in an "if and only if" statement.

Example

Consider the conditional statement "If a triangle is obtuse, then the triangle has one angle between 90 and 180 degrees." Write its converse, and if both are true, write a biconditional statement.

~ "If a triangle has one angle between 90° and 180° , then it is obtuse".

\Rightarrow "If, and only if a triangle has one angle between 90° and 180° , then it is obtuse".

There is one last special conditional statement. If we take the converse of a conditional statement and negate both the hypothesis and the conclusion, we create a "contrapositive" statement.

For example, consider the following conditional statement:

"If a Canadian citizen is 18 years old or older, then that person is able to vote."

Its converse is:

"If a Canadian can vote, then that Canadian is 18 years old or older."

Then its contrapositive statement would be:

"If a Canadian is **not** able to vote, then that Canadian is **not** 18 years old or older."

Example

Write both the converse and contrapositive statements of the following conditional statement:

"If a triangle is obtuse, then the triangle has one angle between 90 and 180 degrees."

Converse:

"If a triangle has an angle between 90° and 180° , then it is obtuse."

Contrapositive:

"If a triangle does not have an angle between 90° and 180° , then it is not obtuse."

Example

Consider the statement "if $z > 2$, then $z^2 > 4$."

Is this conditional statement true? If not, give a counter example.

It is true.

Write the converse of the statement. Is it true? Give a counterexample if it's not.

"If $z^2 > 4$, then $z > 2$ ".

not true. if $z = -3$, then $z^2 = (-3)^2 = 9$, but $-3 \not> 2$.

Write the contrapositive of the original statement. Is it true? Give a counterexample if it's not.

"If $z^2 \not> 4$, then $z \not> 2$."

This is true.

Example

Consider the following conditional statement:

"If Peter lives in ~~Saskatchewan~~ ^{BC}, then Peter lives in ~~Regina~~ ^{Vancouver}."

Is this statement true?

No

Write the converse. Is it true? ^{then}

"If Peter lives in Vancouver, ~~he~~ ^{then} lives in BC."

Yes.

Write the contrapositive of the original statement. Is it true?

"If Peter does not live in Van, then he does not live in BC."

False.

In general,



- If a conditional statement is true, so is its contrapositive.
- If a conditional statement is false, then so is its contrapositive.

