6.2 - Solving Equations Using Balance Strategies

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6.2 Solving Equations by Using Balance Strategies

Consider the following equation:

$$4x + 6 = 7x$$

We can no longer use our strategy of opposite operations because the variable occurs more than once in the equation, and we cannot collect the like terms because they are on the other side of the equal sign.

We need to isolate the variables on one side of the equation, and the constant terms on the other.

Think of the variables as west-coast rappers, like Tupac or Dr. Dre

And think of the constants as east-coast rappers, like Jay-Z or the Notorious B.I.G.

During the 90's, there was a deadly feud between the east and west-coast rap scenes in the US, so we need to keep them on opposite sides of the equal sign.

In this example, for all the variables to be on one side, and all constants on the other, we can move the 4x to the right side of the equation.

The good news is we can use our method of **opposite operations** to move the variables around.

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Because the 4x is **positive**, the opposite operation would be **negative** 4x (ie. Subtract 4x)

So our opposite operation will be: subtraction (Remember, **both sides!**)

$$\cancel{4x} + 6 = 7x$$
$$-\cancel{4x} \quad -4x$$
$$6 = 7x - 4x$$
$$6 = 3x$$

Now we can collect like terms:

$$6 = 3x$$

Now we are left with one variable, and we can use our **opposite operations** as usual:

What operation is placed on the x?

Multiplication

So the opposite operation would be:

Division $\rightarrow$ $\frac{6}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$$x = 6/3 = 2$$

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Example 1: Solving Equations Involving More than One Step

Solve for n: -3n + 7 = 2n - 8

It doesn't matter what side we place the constants or variables on, just as long as they are all completely separated. Let's just decide to place the variables to the right and the constants on the left (in this case, it keeps everything positive).

Let's move the -3n first. What is the opposite operation we will use on both sides to move that term?

$$\cancel{-3n} + 7 = 2n - 8$$
$$\cancel{+3n} \qquad\qquad +3n$$
$$7 = 5n - 8$$

Now let's move the -8 to the left side. What is the opposite operation we will use on both sides to move that term?

Add.

$$7 = 5n \cancel{- 8}$$
$$+8 \qquad\qquad \cancel{+8}$$
$$15 = 5n$$

Now, collect like terms, and use opposite operations to solve as usual:

$$\frac{15}{5} = \frac{\cancel{5}n}{\cancel{5}}$$

$$n = 15/5 = 3$$

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Example 2: Solving Equations Involving Division

Solve for "r": $\frac{110}{r} = 22$

Our goal should be to get rid of the fraction. Our variable needs to be in the numerator (on top) in order for us to solve the equation.

In this case, the operation attached to the variable is division, but the variable *itself* is what we're dividing by.

The **opposite operation** of division is multiplication, but since we multiply both sides by what is being divided, we have to multiply by the variable itself:

$$\cancel{r} \times \frac{110}{\cancel{r}} = 22 \times r$$

$$110 = 22r$$

Notice how we are left with the variable without a fraction. We can now use **opposite operations** as usual:

$$\frac{110}{22} = \frac{\cancel{22}r}{\cancel{22}}$$

$$r = \frac{110^{\div 2}}{22_{\div 2}} = \frac{55}{11} = 5$$

Then, we can check our solution:

$$\frac{110}{r} = 22$$

$$\frac{110}{(5)} \stackrel{?}{=} 22$$

$$22 = 22$$

$$LS = RS!$$

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Example 3: Solving Equations Involving Division

Solve for "a": $\frac{2a}{3} = \frac{4a}{5} + 7$

In this case, we now have 2 fractions, each with a different denominator. Our goal is still to clear the fractions so we can solve for "a".

Just like when we add and subtract fractions, we need to find the least common denominator (LCD).

List the multiples of 3 and 5:

3: 3, 6, 9, 12, 15, 18, 21, ...

5: 5, 10, 15, 20, 25, ...

Therefore, the LCD is: 15

Now, this is the number we will multiply our equation by. Both sides, of course:

$$15 \times \left[\frac{2a}{3}\right] = \left[\frac{4a}{5} + 7\right] \times 15$$

We will need to use the distributive property on the right side:

$$\frac{\cancel{15}^{5} \times 2a}{\cancel{3}} = \frac{\cancel{15}^{3} \times 4a}{\cancel{5}} + 15 \times 7$$

$$5 \times 2a = 3 \times 4a + 15 \times 7$$

$$10a = 12a + 105$$

Notice how there are no longer fractions in the equation. We now use our previous strategies to solve for "a":

$$10a = \cancel{12a} + 105$$

$$-12a \quad -\cancel{12a}\ 0$$

$$\frac{-\cancel{2}^{1}a}{-\cancel{2}} = \frac{105}{-2}$$

$$a = \frac{105}{-2} = -52.5$$

Textbook Assignment: Pg. 281 # 7 – 13, 19*, 22*